Testing Nesting DXTRAN Spheres for Neutron Transport in Air

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M. L. Fensin, K. C. Kelley, J. S. Hendricks,
B. W. Cox, J. T. Goorley and S. S. McCready

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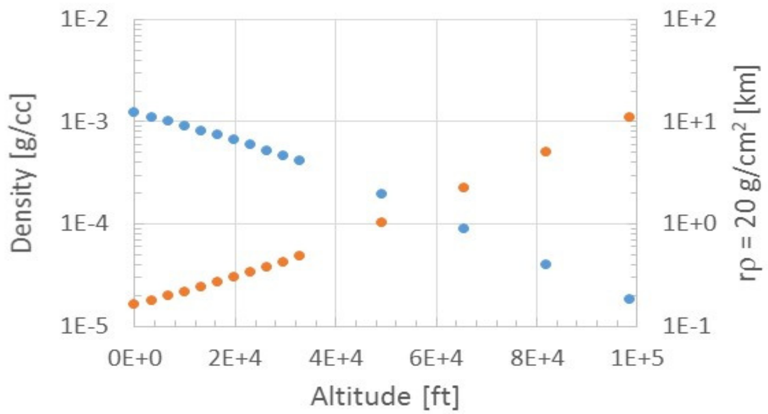
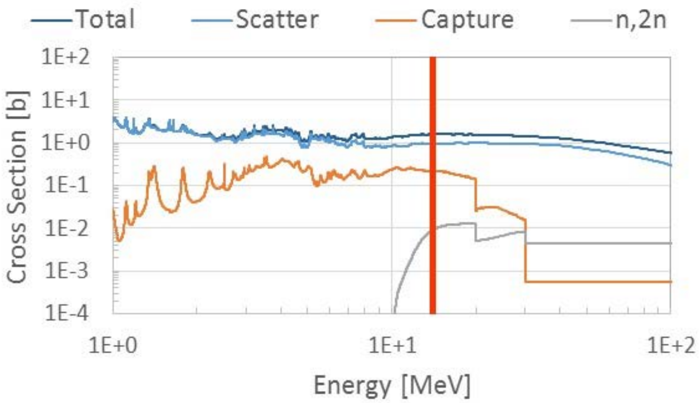
Outline

- Transport Considerations for Neutron Transport in Air
- Statistics
- VR Feature Descriptions
- Results
- Conclusions

Neutron Transport in Air

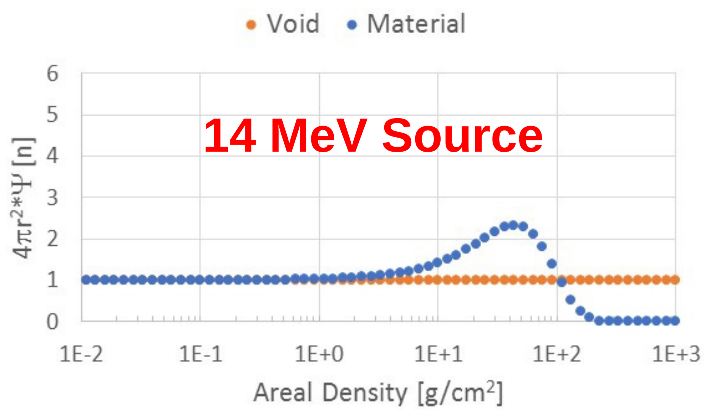
- **Neutron transport in air for large standoff detection**
  - Small solid angle
  - Scatter dominates
- **Deterministic**
  - Ray effects
- **Monte Carlo**
  - Analog is extremely inefficient
  - Variance Reduction required to get particles to a detector

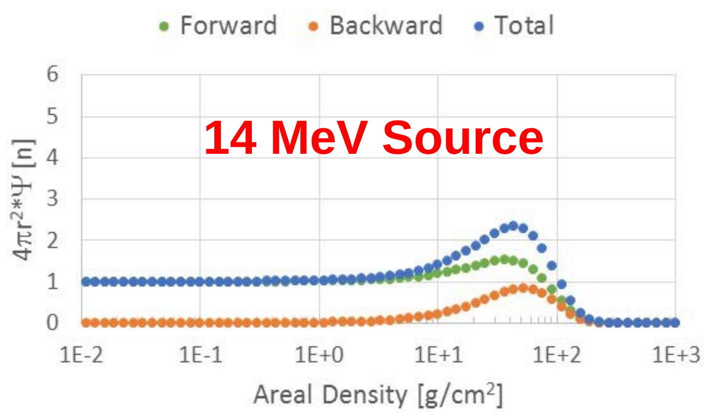




Transport Considerations

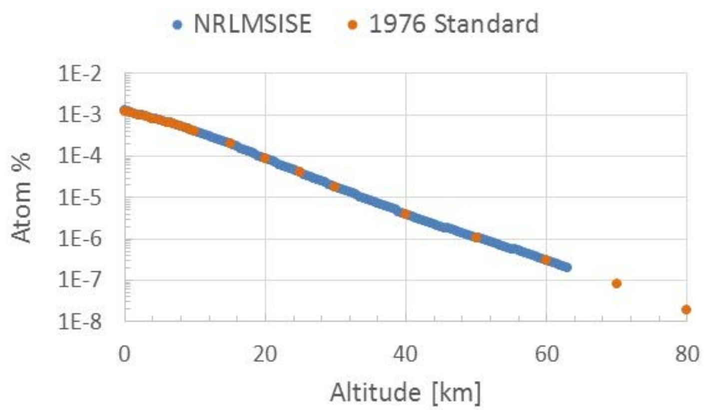
- **Surface fluence of isotropic point source emitted in a void ~1/r²**
  - $4\pi r^2 \Psi(\vec{r})$ is therefore independent of r in a void
- **Departure from 1/r² is related to scatter and capture**
  - Buildup peak between starts ~10 g/cm²

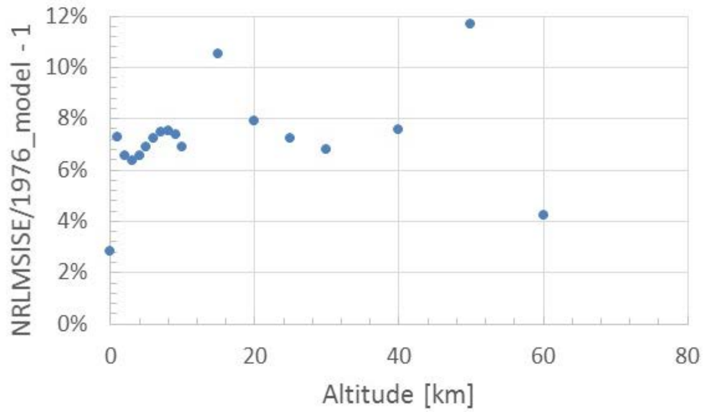




Geometry

- **Atmosphere model**
  - NRLMSISE-00 (**N**aval **R**esearch **L**aboratory **M**ass **S**pectrometer and **I**ncoherent **S**catter radar **E**xtends)-- Community Coordinated Modeling Center at Goddard Space Flight Center (2/2/2016)
    - Extends the ground through exosphere and 00 is the year of the release
  - 60 X 60 X 60 km
  - 0.5 km altitude increments
  - Nevada ground
  - **N-14, O-16, Ar-40 and He-4**
- **14 MeV isotropic point source**
  - 50k ft (~15.24 km) in altitude,1 km from 3 m radius detector





Concentration Assumptions

- **Prior Analysis**
  - Assuming only N-14, O-16, **He-4** and Ar-40 is reasonable
- **Assumed Concentration (at. %)**
  - N-14 78.11%
  - O-16 20.1%
  - He-4 0.001%
  - Ar-40 0.934%

Areal Density	Elastic Scatter Contribution			
[g/cm^2]	N-14	O-16	Ar-40	Total
20	81.5%	18.0%	0.3%	99.7%
40	83.9%	15.6%	0.2%	99.7%
60	85.1%	14.4%	0.2%	99.7%
80	85.8%	13.8%	0.1%	99.8%
100	86.3%	13.4%	0.1%	99.8%

Areal Density	Inelastic Scatter Contribution			
[g/cm^2]	N-14	O-16	Ar-40	Total
20	73.3%	21.8%	4.1%	99.2%
40	72.8%	22.2%	4.3%	99.2%
60	72.6%	22.5%	4.2%	99.3%
80	72.4%	22.9%	4.1%	99.4%
100	72.5%	23.0%	3.9%	99.4%

Central Limit Theorem

- **Estimated mean approaches a normal distribution with known variance as the number of random samples grows sufficiently large**
- **Mean and Variance must exist**
  - $E(x) = \mu = \int_{-\infty}^{\infty} x f(x) dx = true\ mean$
  - $E(x^2) = \sigma^2 = \int_{-\infty}^{\infty} \left(x - E(x)\right)^2 f(x) dx$
- **Mean and Variance must be calculated**
  - $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$
  - $S^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}$
- **How does one determine if the variance is finite, in order to meet the existence requirement?**

Slope Test

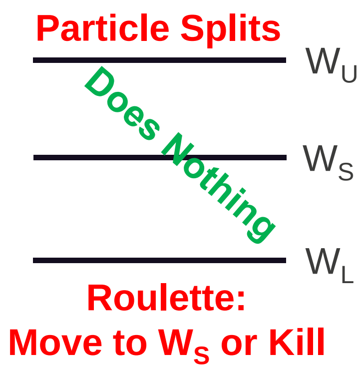
- **Fit f(x) to a known function using “simplex” algorithm**
  - Fit the top 201 history scores
  - $\boldsymbol{f(x) = \dfrac{\left(1+\frac{kx}{a}\right)^{-\left(\frac{1}{k}+1\right)}}{a}}$
  - Slope = (1/k)+1
- $\boldsymbol{E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx}$
  - If slope is not greater than 3, enough histories may not have yet been sampled

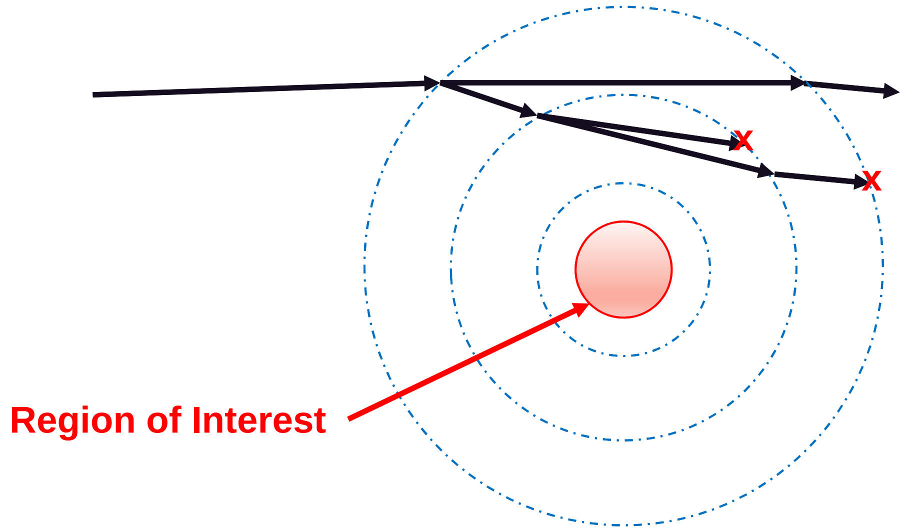
```
1unnormed tally density for tally 2         nonzero tally mean(m) = 1.722E-05   nps = 10000000            print table 161

 abscissa              ordinate   log plot of tally probability density function in tally fluctuation chart bin(d=decade,slope= 3.1)
  tally  number num den log den:d--------d---------d--------d---------d---------d---------d---------d--------d---------d---------d--
 6.31-07      12 9.25+00   0.966 *********|*********|********|*********|*********|*******  |         |        |         |         |
 7.94-07     416 2.55+02   2.406 *********|*********|********|*********|*********|*********|*********|*       |         |         |
 1.00-06   35076 1.71+04   4.232 *********|*********|********|*********|*********|*********|*********|********|*********|         |
 1.26-06   32167 1.24+04   4.094 *********|*********|********|*********|*********|*********|*********|********|******** |         |
 1.58-06 1.0+06 3.15+05   5.498 *********|*********|********|*********|*********|*********|*********|********|*********|*********|**
 2.00-06 881910 2.15+05   5.332 *********|*********|********|*********|*********|*********|*********|********|*********|*********|
 2.51-06 784963 1.52+05   5.182 *********|*********|********|*********|*********|*********|*********|********|*********|*********|
 3.16-06 944860 1.45+05   5.162 *********|*********|********|*********|*********|*********|*********|********|*********|*********|
 3.98-06 897382 1.10+05   5.040 *********|*********|********|*********|*********|*********|*********|********|*********|******** |
 5.01-06 841760 8.17+04   4.912 *********|*********|********|*********|*********|*********|*********|********|*********|******   |
 6.31-06 751717 5.79+04   4.763 *********|*********|********|*********|*********|*********|*********|********|*********|*****    |
 7.94-06 646037 3.95+04   4.597 *********|*********|********|*********|*********|*********|*********|********|*********|***      |
 1.00-05 538158 2.62+04   4.418 *********|*********|********|*********|*********|*********|*********|********|*********|*        |
 1.26-05 439076 1.70+04   4.229 *********|*********|********|*********|*********|*********|*********|********|*********|         |
 1.58-05 353061 1.08+04   4.035 *********|*********|********|*********|*********|*********|*********|********|******** |         |
 2.00-05 285399 6.95+03   3.842 mmmmmmmmm|mmmmmmmmm|mmmmmmmm|mmmmmmmmm|mmmmmmmmm|mmmmmmmmm|mmmmmmmmm|mmmmmmmm|mmmmmm   |         |
 2.51-05 231660 4.48+03   3.652 *********|*********|********|*********|*********|*********|*********|********|****     |         |
 3.16-05 189579 2.91+03   3.465 *********|*********|********|*********|*********|*********|*********|********|**       |         |
 3.98-05 156969 1.92+03   3.283 *********|*********|********|*********|*********|*********|*********|********|         |         |
 5.01-05 132501 1.29+03   3.109 *********|*********|********|*********|*********|*********|*********|********|         |         |
 6.31-05 111775 8.61+02   2.935 *********|*********|********|*********|*********|*********|*********|******  |         |         |
 7.94-05  96234 5.89+02   2.770 *********|*********|********|*********|*********|*********|*********|****    |         |         |
 1.00-04  81951 3.98+02   2.600 *********|*********|********|*********|*********|*********|*********|***     |         |         |
 1.26-04  70354 2.72+02   2.434 *********|*********|********|*********|*********|*********|*********|*       |         |         |
 1.58-04  58697 1.80+02   2.255 *********|*********|********|*********|*********|*********|*********|        |         |         |
 2.00-04  48262 1.18+02   2.070 *********|*********|********|*********|*********|*********|*******  |        |         |         |
 2.51-04  38762 7.50+01   1.875 *********|*********|********|*********|*********|*********|******   |        |         |         |
 3.16-04  29566 4.55+01   1.658 *********|*********|********|*********|*********|*********|***      |        |         |         |
 3.98-04  21972 2.68+01   1.429 *********|*********|********|*********|*********|*********|*        |        |         |         |
 5.01-04  16072 1.56+01   1.193 *********|*********|********|*********|*********|*********|         |        |         |         |
 6.31-04  11477 8.84+00   0.947 *********|*********|********|*********|*********|******   |         |        |         |         |
 7.94-04   7945 4.86+00   0.687 *********|*********|********|*********|*********|****     |         |        |         |         |
 1.00-03   5223 2.54+00   0.405 *********|*********|********|*********|*********|*        |         |        |         |         |
 1.26-03   3265 1.26+00   0.101 *********|*********|********|*********|******** |         |         |        |         |         |
 1.58-03   1745 5.35-01  -0.271 *********|*********|********|*********|*****    |         |         |        |         |         |
 2.00-03    922 2.25-01  -0.648 *********|*********|********|*********|*        |         |         |        |         |         |
 2.51-03    395 7.65-02  -1.117 *********|*********|********|******   |         |         |         |        |         |         |
 3.16-03    153 2.35-02  -1.628 *********|*********|********|*        |s        |         |         |        |         |         |
 3.98-03     58 7.08-03  -2.150 *********|*********|*****   |        s|         |         |         |        |         |         |
 5.01-03     31 3.01-03  -2.522 *********|*********|*       |      s  |         |         |         |        |         |         |
 6.31-03     14 1.08-03  -2.967 *********|*******  |        |    s    |         |         |         |        |         |         |
 7.94-03      9 5.51-04  -3.259 *********|****     |        | s       |         |         |         |        |         |         |
 1.00-02      1 4.86-05  -4.313 ****     |         |        s         |         |         |         |        |         |         |
 1.26-02      4 1.54-04  -3.811 *********|         |     s  |         |         |         |         |        |         |         |
 1.58-02      1 3.07-05  -4.513 **       |         |   s    |         |         |         |         |        |         |         |
 2.00-02      1 2.44-05  -4.613 *        |         |s       |         |         |         |         |        |         |         |
 2.51-02      2 3.87-05  -4.412 ***      |       s |        |         |         |         |         |        |         |         |
  total 9774215 9.77-01         d--------d---------d--------d---------d---------d---------d---------d--------d---------d---------d--
```

Trouble with WW’s

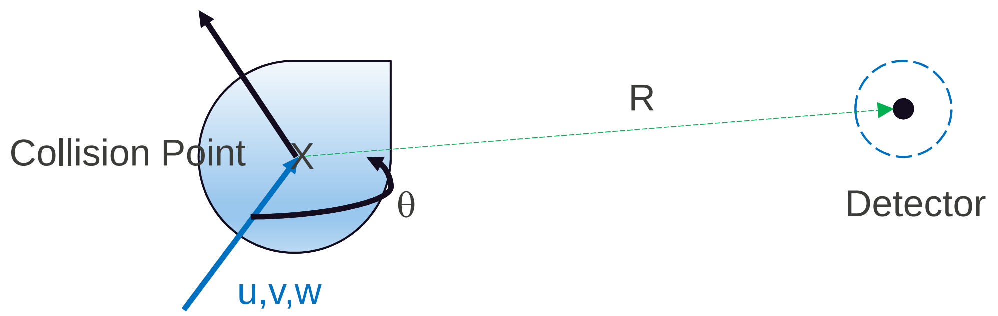
- **WWs split/roulette particles at surface crossings and collisions**
- **WWs cannot focus particles towards a finite detector**





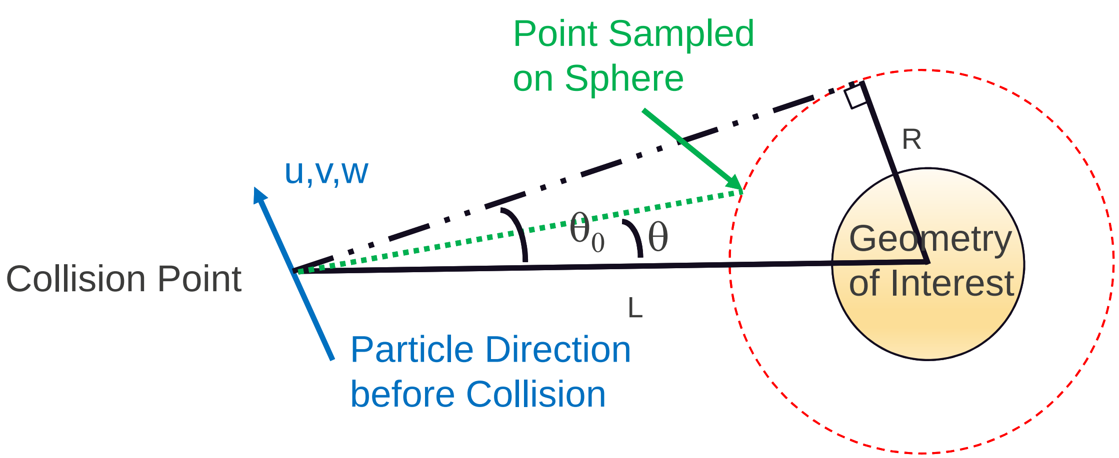
Deterministic Estimates

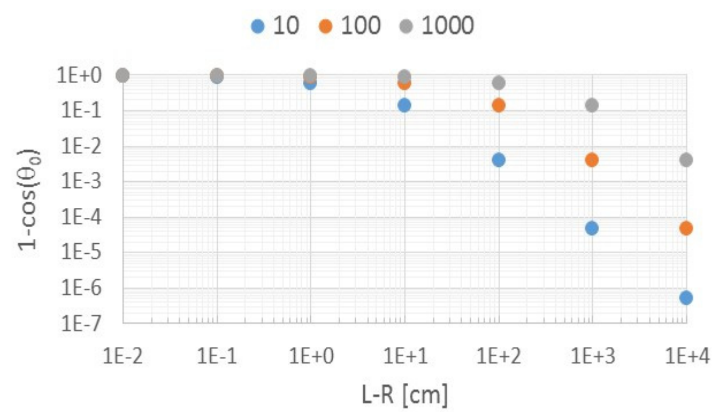
- **Point detectors place a source or collision**
  - $\Phi = \frac{Wp(\mu)e^{-\lambda}}{2\pi R^2}$
  - Large weight fluctuation can occur because of $1/R^2$,
  - *The point detector estimate is assumed to be the average flux uniformly distributed within the radius of exclusion*



DXTRAN

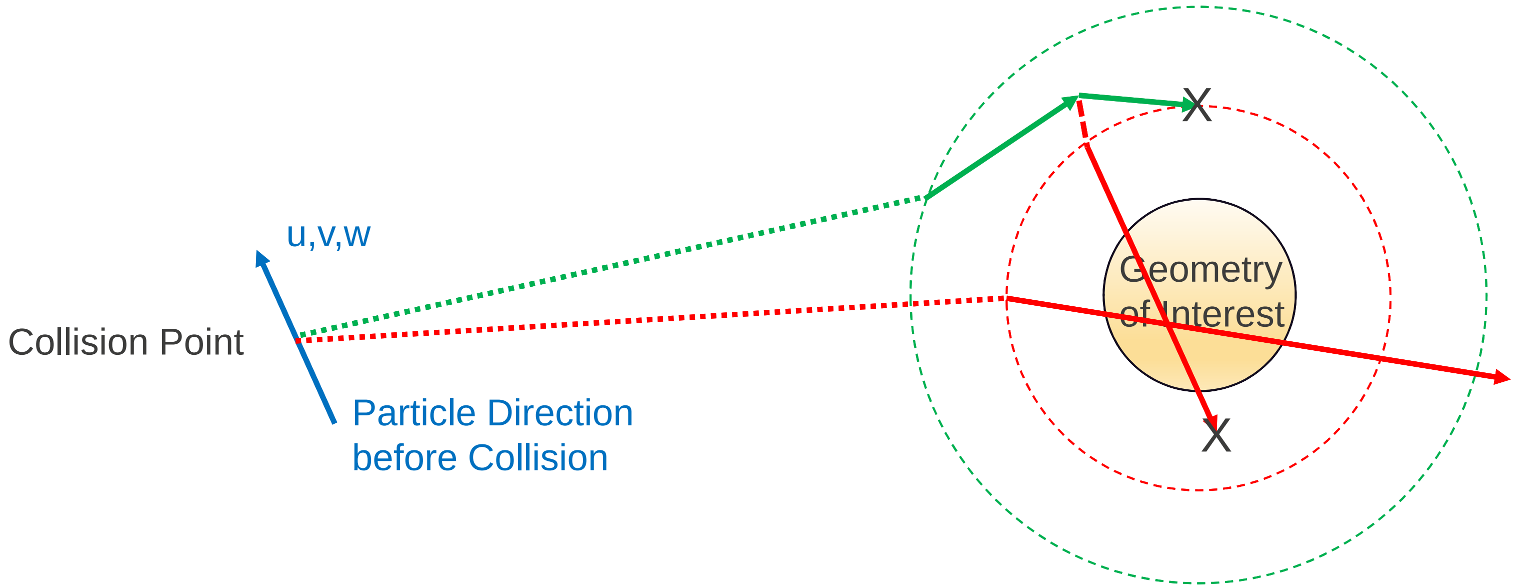
- **At a collision event, a DXTRAN particle is placed on the sphere**
  - $\cos(\theta_0) = \frac{\sqrt{L^2 - R^2}}{L}$
    - $\cos(\theta) = \cos(\theta_0) + \xi(1 - \cos(\theta_0))$
  - $w = w_0 P(\cos(\theta))(1 - \cos(\theta_0))e^{-\int \sigma_t(s)ds}$
  - To balance weight, NONDXTRAN particles are killed when traversing the sphere





Nested DXTRAN

- **Nested DXTRAN sphere concept was devised to control this large weight fluctuation**
  - dxt X_1 Y_1 Z_1 RI_1 RO_1…X_N Y_N Z_N RI_N RO_N DWC_1 DWC_2 DPWT



Single DXTRAN sphere test (ex1)

```
dxt:n 0 0 1524000 310 310
```

- **A simple DXTRAN sphere 310 cm radius centered around tally**
  - dwc1 and dwc2 not set
  - 10 million histories
- **Did not pass all 10 statistical checks**

Case	Mean	Relative Error	VOV	Slope	FOM
ex1	1.4804E-11	0.0081	0.1263	2.3	163

Single DXTRAN sphere test (ex2)

```
average weight per history = 4.51829E-06        largest weight = 2.28798E-01
largest/average = 5.06383E+04                   nps of largest =     2033847
```

```
dxt:n 0 0 1524000 310 310
     4.5e-6 4.5e-7
```

dwc's apply to particles in RI
Passed all 10 statistical checks

Case	Mean	Relative Error	VOV	Slope	FOM
ex1	1.4804E-11	0.0081	0.1263	2.3	163
ex2	1.4589E-11	0.0049	0.0305	3.3	449

ex2/ex1-1= -1.45%

DD cards

- **Roulette in transmission**
  - The average contribution to a DXTRAN sphere is calculated from all contributions to the sphere made by particle histories until the first TFC interval is reached → moving target. Then the average contribution is updated at each TFC rendezvous
- **DD card sets the roulette/survival weight for transmission**
- **DDn k_1 m_1 k_2 m_2 …**
  - n = 1 for n spheres; 2 for photon spheres
  - k → If negative, if greater than -1*k, keep particle; else roulette
  - m → print first 100 contributions greater than -1*m*k

Examine TFC (ex3)

```
dxt:n 0 0 1524000 310 310
     4.5e-6 4.5e-7
dd1 -4.5e-7 1e4
```

Failed some statistical checks!!!
Examine VOV

Case	Mean	Relative Error	VOV	Slope	FOM
ex1	1.4804E-11	0.0081	0.1263	2.3	163
ex2	1.4589E-11	0.0049	0.0305	3.3	449
ex3	1.4684E-11	0.0065	0.1527	2.7	258

ex3/ex1-1 < +/-1%

nps	mean	error	vov	slope	fom
512000	1.5507E-11	0.0283	0.2033	2.4	263
1024000	1.4828E-11	0.0167	0.1314	2.4	380
1536000	1.4555E-11	0.0121	0.1016	2.6	480
2048000	1.4499E-11	0.0099	0.0739	2.8	535
2560000	1.4533E-11	0.0089	0.0515	2.5	533
3072000	1.4539E-11	0.0080	0.0399	2.6	546
3584000	1.4584E-11	0.0076	0.0318	2.7	530
4096000	1.4587E-11	0.0070	0.0270	2.8	546
4608000	1.4639E-11	0.0071	0.0549	2.6	460
5120000	1.4655E-11	0.0067	0.0469	3.1	470
5632000	1.4689E-11	0.0072	0.0660	2.5	370
6144000	1.4722E-11	0.0068	0.0583	2.6	381
6656000	1.4849E-11	0.0089	0.2050	2.5	204

3 Nested DXTRAN spheres – ex4 and ex5

```
dxt:n 0 0 1524000 310 310
      0 0 1524000 620 620
      0 0 1524000 1240 1240
      4.5e-6 4.5e-7
```

Low FOM
Failed some statistical checks!!!

```
dxt:n 0 0 1524000 310 310
      0 0 1524000 620 620
      0 0 1524000 1240 1240
      4.5e-6 4.5e-7
dd1  -4.5e-7 1e4 -1.79e-6 1e4
     -7.2e-6 1e4
```

Failed some statistical checks!!!

Case	Mean	Relative Error	VOV	Slope	FOM
ex1	1.4804E-11	0.0081	0.1263	2.3	163
ex2	1.4589E-11	0.0049	0.0305	3.3	449
ex3	1.4684E-11	0.0065	0.1527	2.7	258
ex4	1.4942E-11	0.0080	0.5386	2.8	67
ex5	1.4807E-11	0.0039	0.0162	2.7	274

ex4/ex1-1 < +/-1%
ex5/ex1-1 < +/-1%

Try WWs (ex6-6c)

```
dxt:n 0 0 1524000 310 310
      0 0 1524000 620 620
      0 0 1524000 1240 1240
dd1  -4.5e-7 1e4 -1.79e-6 1e4
     -7.2e-6 1e4
C wwp:n 4j -1
wwg 2 0
mesh  geom=sph ref=-100000 0 1524000
      origin 0 0 1524000
      axs=1 0 0 vec=0 1 0
      imesh 310 620 1240 1600000
      iints 2 2 2 14
      jmesh 0.5
      jints 1
      kmesh 1
      kints 1
```

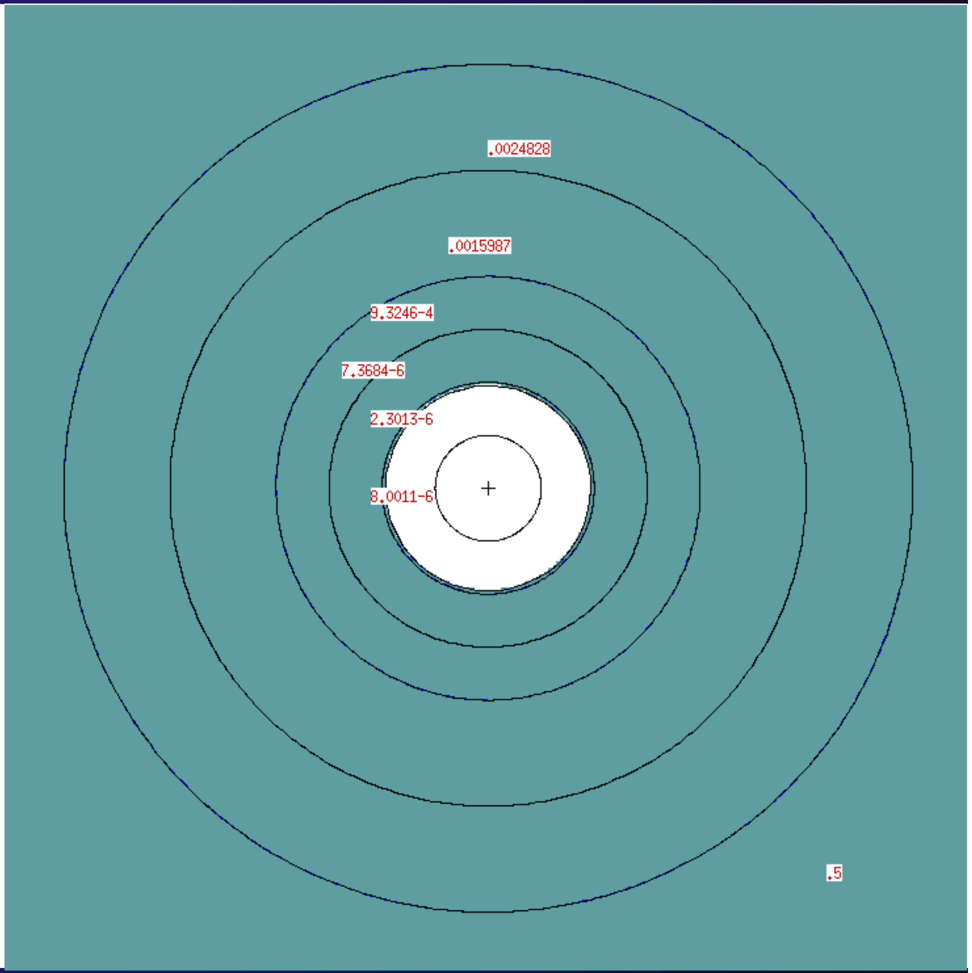
0th and 1st 1e5, 2nd 1e6, 3rd 1e7
0th passed? 3rd passed – mean is higher!!!

Case	Mean	Relative Error	VOV	Slope	FOM
ex2	1.4589E-11	0.0049	0.0305	3.3	449
ex6	1.4111E-11	0.0219	0.0413	3.2	840
ex6a	1.4732E-11	0.0328	0.0785	2.7	1112
ex6b	1.5344E-11	0.0248	0.2609	2.6	180
ex6c	1.4877E-11	0.0052	0.0116	4.3	416

ex6/ex1-1 = -4.68%
ex6a/ex1-1 < +/-1%
ex6b/ex1-1 = 3.65%
ex6c/ex1-1 < +/-1%

Examine WWs

- **WW's varied by 2 orders of magnitude in certain regions**
  - Inefficient?



Better WWs? (ex7-7c)

```
dxt:n 0 0 1524000 310 310
      0 0 1524000 620 620
      0 0 1524000 1240 1240
dd1 -4.5e-7 1e4 -1.79e-6 1e4 -7.2e-6 1e4
wwp:n 4j -1
wwg 2 0
mesh  geom=sph ref=-100000 0 1524000
      origin 0 0 1524000
      axs=1 0 0 vec=0 1 0
      imesh 311 621 1241 1e4 1e5 1e6
      1600000
      iints 1 6 5 9 9 9 15
      jmesh 0.5 jints 1 kmesh 1 kints 1
```

Case	Mean	Relative Error	VOV	Slope	FOM
ex6c	1.4877E-11	0.0052	0.0116	4.3	416
ex7	1.4111E-11	0.0219	0.0413	3.2	862
ex7a	1.4691E-11	0.0178	0.0309	3	2597
ex7b	1.5004E-11	0.0085	0.1455	2.8	1100
ex7c	1.4914E-11	0.0023	0.0097	3.3	1502

FOM went up!

ex7/ex1-1 = -4.68%
ex7a/ex1-1 < +/-1%
ex7b/ex1-1 = 1.35%
ex7c/ex1-1 < +/-1%

What about 6 spheres? (ex8-8d)

```
dxt:n 0 0 1524000 310 310
      0 0 1524000 700 700
      0 0 1524000 1560 1560
      0 0 1524000 3400 3400
      0 0 1524000 7500 7500
      0 0 1524000 16000 16000 1e-2 1e-7
dd1   -4.50E-07 1e4 -2.30E-06 1e4
      -1.14E-05 1e4 -5.40E-05 1e4
      -2.60E-04 1e4 -1.23E-03 1e4
wwp:n 4j -1
wwg 2 0
mesh  geom=sph ref=-100000 0 1524000
      origin 0 0 1524000
      axs=1 0 0 vec=0 1 0
      imesh 311 621 1241 1e4 1e5 1e6
      1600000
      iints 1 6 5 9 9 9 15
      jmesh 0.5  jints 1
      kmesh 1 kints 1
```

Is this a bad idea?

Case	Mean	Relative Error	VOV	Slope	FOM
ex7c	1.4914E-11	0.0023	0.0097	3.3	1502
ex8	1.4781E-11	0.0208	0.3914	3.2	500
ex8a	1.4659E-11	0.0200	0.4735	4.7	1015
ex8b	1.4857E-11	0.0200	0.4501	4.1	1035
ex8c	1.4849E-11	0.0066	0.1137	2.2	910
ex8d	1.4860E-11	0.0023	0.0236	1.7	785

0th, 1st and 2nd 1e5 histories, 3rd 1e6 histories, 4th 1e7 histories

ex8/ex1-1 < +/-1%
ex8a/ex1-1 < +/-1%
ex8b/ex1-1 < +/-1%
ex8c/ex1-1 < +/-1%
ex8d/ex1-1 < +/-1%

Use a point detector to generate WWs? (ex9-9d)

```
dxt:n 0 0 1524000 310 310
      0 0 1524000 700 700
      0 0 1524000 1560 1560
      0 0 1524000 3400 3400
      0 0 1524000 7500 7500
      0 0 1524000 16000 16000 1e-2 1e-7
dd1   -4.50E-07 1e4 -2.30E-06 1e4
      -1.14E-05 1e4 -5.40E-05 1e4
      -2.60E-04 1e4 -1.23E-03 1e4
f5:n 0 0 1524000 1
wwp:n 4j -1
wwg 5 0
mesh  geom=sph ref=-100000 0 1524000
      origin 0 0 1524000
      axs=1 0 0 vec=0 1 0
      imesh 311 621 1241 1e4 1e5 1e6
      1600000
      iints 1 6 5 9 9 9 15
      jmesh 0.5  jints 1
      kmesh 1 kints 1
```

Case	Mean	Relative Error	VOV	Slope	FOM
ex7c	1.4914E-11	0.0023	0.0097	3.3	1502
ex9	1.4410E-11	0.0122	0.0061	5.7	1178
ex9a	1.4699E-11	0.0157	0.1940	4.7	1412
ex9b	1.4256E-11	0.0186	0.4387	3.4	1021
ex9c	1.4824E-11	0.0061	0.1463	2.3	967
ex9d	1.4997E-11	0.0044	0.1327	1.7	182

Hmmmm?

ex9/ex1-1 < -2.66%
ex9a/ex1-1 < +/-1%
ex9b/ex1-1 < 3.70%
ex9c/ex1-1 < 0.14%
ex9d/ex1-1 < 1.3%

What is going on

- **A 14 MeV neutron has a MFP of ~770 m at 50,000 ft; therefore the source placement was less than 2 MFP's from the tally**
  - As a result of distance, solid angle from source to tally was still really small
- **Sampling enough histories eventually resulted in sampling a collision in the vicinity of the DXTRAN sphere, which caused a larger weight score (view factor close to 1) that spiked the VOV and reduced the slope**
- **In cases ex6 and ex7, particles were pushed closer to the DXTRAN sphere as a result of splitting through the weight window**
  - Collisions cannot be guaranteed as a result of WW splitting; and therefore due to large MFP of the air in the vicinity of the DXTRAN sphere, a particle will only rarely collide and contribute to the surface tally

Use fcl? (ex10-10d)

```
dxt:n 0 0 1524000 310 310
      0 0 1524000 700 700
      0 0 1524000 1560 1560
      0 0 1524000 3400 3400
      0 0 1524000 7500 7500
      1e-2 1e-7
dd1   -4.50E-07 1e4 -2.30E-06 1e4
      -1.14E-05 1e4 -5.40E-05 1e4
      -2.60E-04 1e4
wwp:n j j j j -1 J J
wwg 1 0 j J J J J j
mesh  geom=sph ref=-100000 0 1524000
      origin 0 0 1524000
      axs=1 0 0 vec=0 1 0
      imesh 311 701 1561 3401 7501 16001
      1e5 1e6 1600000
      iints 1 6 6 6 6 9 9 9  15
      jmesh 0.5 jints 1
      kmesh 1  kints 1
```

Case	Mean	Relative Error	VOV	Slope	FOM
ex7c	1.4914E-11	0.0023	0.0097	3.3	1502
ex10	1.4742E-11	0.0144	0.0114	5.5	828
ex10a	1.4763E-11	0.0120	0.0045	5.7	1403
ex10b	1.4838E-11	0.0119	0.0032	10	1420
ex10c	1.4889E-11	0.0038	0.0006	4.6	1384
ex10d	1.4878E-11	0.0012	0.0004	3.1	1344

ex10n/ex1-1 < +/-1%

902 32 -1.991E-04 901 -902 imp:n=1 fcl:n=0.7
903 32 -1.991E-04 902 -903 imp:n=1 fcl:n=0.8
904 32 -1.991E-04 903 -904 imp:n=1 fcl:n=0.9
905 32 -1.991E-04 904 -905 imp:n=1 fcl:n=1

VOV not decreasing by 1/N for ex10a and ex10d; however, this may be to small VOV.

Conclusions

- **At 50,000 ft, the areal density is ~20 g/cm^2**
  - Within buildup region → 1.48E-11*4*π*r^2 **= ~1.86 n** → 1-D model had **~1.86 n** at ~20 g/cm^2
- **DXTRAN requires weight window bounds**
  - Setting dwc1 to the mean and dwc2 to 10% of mean sped up by a factor of 2.75
  - DD cards roulette in transmission
    - Called before dwc's or WW's
    - May help set different weight bounds for different spheres, but may not help so significantly if few MFP's away
- **1 DXTRAN resulted in "false" convergence – but was within <2%**
- **Nested spheres benefit from WW's if spheres are spaced far enough**
  - If chosen strategically WW's can speed calculation by a factor of 3-4
- **Forced collision biasing in the vicinity of the DXTRAN sphere may be useful**
  - But running 3 spheres without fcl still outperformed 6 spheres with fcl
  - For many of the calculations that did not pass all 10 statistical checks the 1 std. confidence interval bounded the correctly converged result
- **What about $RI \neq R0$, or what about high survival weight?**

Future Work

```
dxt:n 0 0 1524000 310 1e3
      0 0 1524000 5e3 1e4 1.0e-2 1e-6
dd1
      -4.7e-6 1e4
      -4.7e-4 1e4
mesh  geom=sph ref=-100000 0 1524000
      origin 0 0 1524000
      axs=1 0 0 vec=0 1 0
      imesh 311 1001 10001 1e5 1e6 1.6e7
      iints 1 6 9 9  9 15
      jmesh 0.5  jints 1
      kmesh 1    kints 1
```

Case	Mean	Relative Error	VOV	Slope	FOM
ex40	1.4786E-11	0.0039	0.0191	6.9	855
ex43	1.4816E-11	0.0109	0.0649	2.6	1000
ex43a	1.4958E-11	0.0121	0.1188	2.4	1849
ex43b	1.4914E-11	0.0042	0.0295	5.6	1461

ex40/ex1-1 < +/-1%

ex43b/ex1-1 < +/-1%

ex40, ex43, and ex43a used 1e6 histories
ex43b used 1e7 histories

If I choose dwc1=1e-5, peformance increases by a factor of 4 but slope drops to 3.1 (f(x) seems like a good fit)... In ex40 and ex43, reported slope seems to be a bad fit when comparing to f(x).