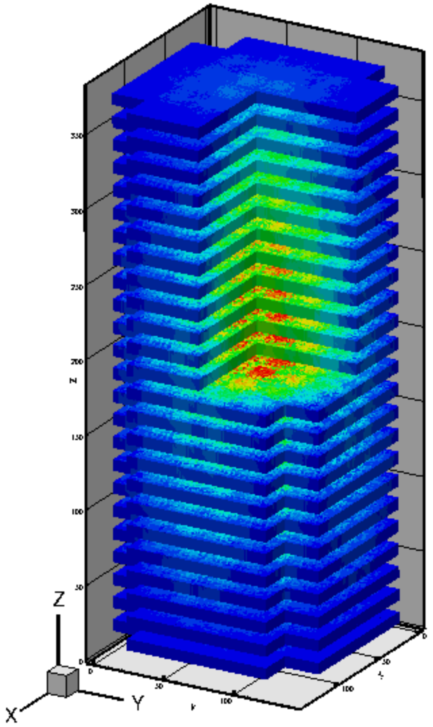
# Evaluation of RAPID for a UNF cask benchmark problem




**Valerio Mascolino, Alireza Haghighat, and Nathan Roskoff**
Virginia Tech
Nuclear Science and Engineering Laboratory
Nuclear Engineering Program, Mechanical Engineering Department
Arlington, VA
http://nsel.ncr.vt.edu

# Outline

- Purpose
- The RAPID code system
- GBC-32 cask computational benchmark
  - System description
  - Establishment of MCNP reference models
  - Comparison of RAPID to MCNP reference models
- Concluding remarks and future work

# Purpose

- Benchmarking of the RAPID Multi-stage Response-function Transport (MRT) code system against the GBC-32 Cask system through comparison with MCNP reference models.

# The RAPID (Real-time Analysis for Particle transport and In-situ Detection) code system



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# MRT

▷ The RAPID code system is developed based on the MRT (*Multi-stage Response-function Transport*) methodology; the MRT methodology is described as follows:

1. *Partition a problem into stages*
2. *Represent each stage by a response function or set of response coefficients*
3. *Pre-calculate response functions and/or coefficients (one time)*
4. *Couple stages through a set of linear system of equations*
5. *Solve the linear system of equations iteratively in real-time*

# The RAPID Code System

- RAPID is capable of calculating the system *eigenvalue* $k_{eff}$, *3D (pin-wise & axially-dependent) fission density distribution, and detector response.*

- RAPID is comprised of **five stages**:

  Pre-calculation (one time)

  Stage 1: Calculation of fission matrix (FM) coefficients, and generation of a database

  Stage 2: Calculation detector field-of-view (FOV) and importance function database

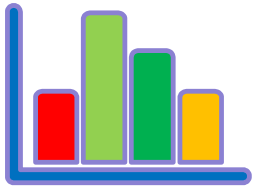
  Calculation

  Stage 3: Processing of FM coefficients

  Stage 4: Solving a linear system of equations, i.e., **Fission Matrix (FM) formulation**

  Stage 5: Detector response calculation

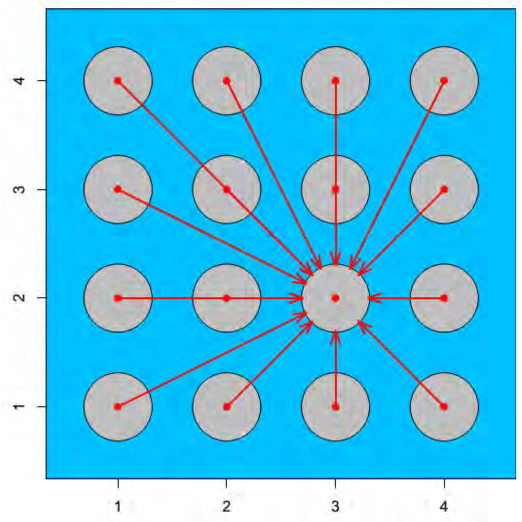
# Fission Matrix (FM) formulation



## Eigenvalue formulation

$$S_i = \frac{1}{k} \sum_{j=1}^{N} a_{i,j} S_j$$

$k$ is eigenvalue
$S_j$ is fission source
$a_{i,j}$ is the number of fission neutrons produced in cell $i$ due to a fission neutron born in cell $j$.




## Subcritical multiplication formulation

$$S_i = \sum_{j=1}^{N} (a_{i,j} S_j + b_{i,j} S_j^{intrinsic})$$

$b_{i,j}$ is the number of fission neutrons produced in cell $i$ due to a source neutron born in cell $j$.

# The GBC-32 cask computational benchmark

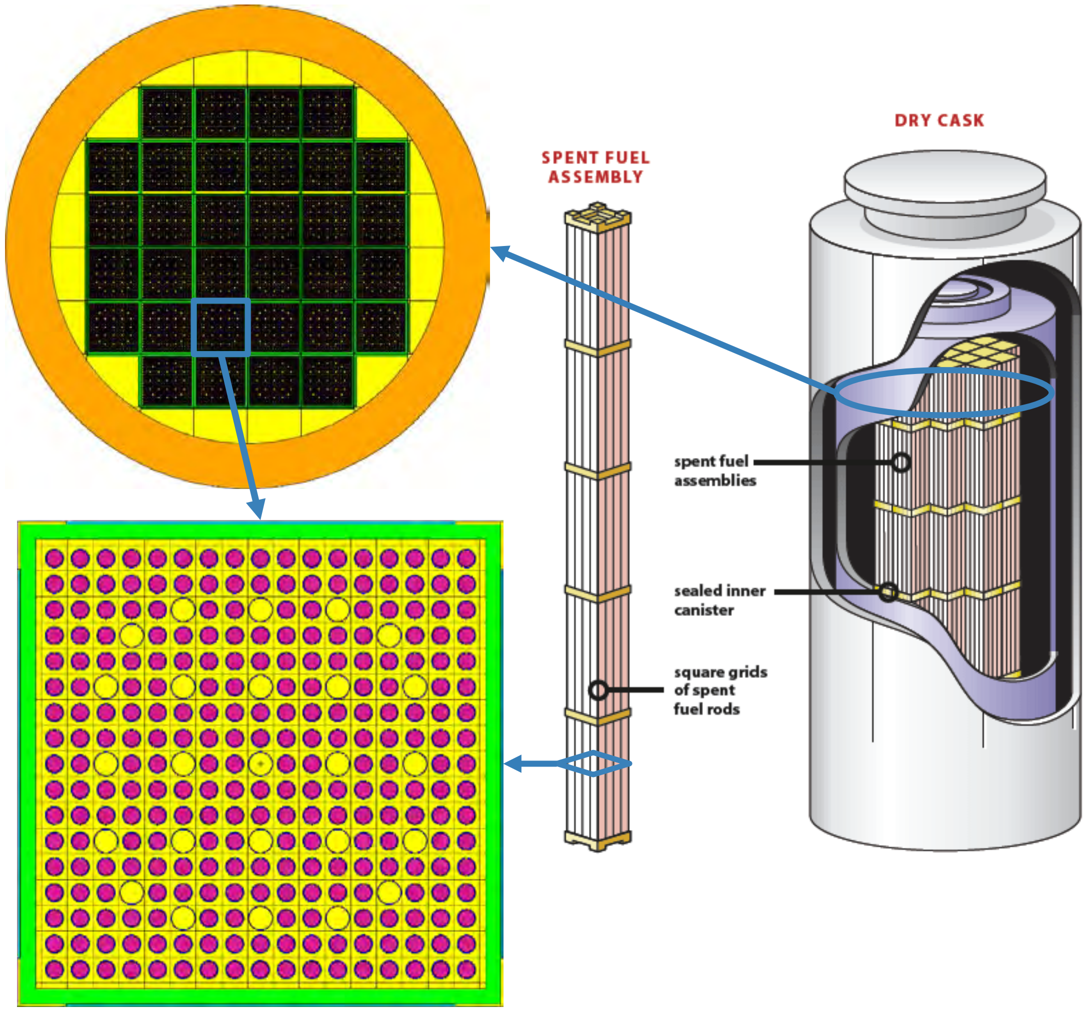
# GBC-32 cask computational benchmark

- **Geometry**

32 Fuel assemblies
Stainless steel (SS304) cylindrical canister
Inter-assembly Boral absorber panels
Height of the canister: 470.76 cm

- **Fuel assembly**

17x17 Optimized Fuel Assembly (OFA)
25 instrumentation guides
<u>Fresh</u> $UO_2$ 4% wt. enriched fuel pins
Active height: 365.76 cm



# Establishment of a reference MCNP model

▷ A reference MCNP model has been established for comparison with the RAPID results

▷ This has been accomplished by examining the convergence of the fission source distribution for a single-assembly model by:

- Parametric analysis of MCNP eigenvalue parameters:
  NSK - Number of Skipped Cycles (NSK)
  NAC - Number of Active Cycles (NAC)
  NPS - Number of Particles per Cycle (NPS)

- Cycle-to-cycle correlation analysis

▷ The results of the single-assembly analysis have been extended to the full cask

# Known eigenvalue Monte Carlo difficulties

- **Source convergence:** Used Nuclear Fuel (UNF) pools or casks due to the presence of absorbers, suffer from undersampling that may result in a biased solution.

- **Cycle -to-cycle correlation:** previous generation is used as source in the power-iteration method, correlation may take places between successive cycles. Statistical uncertainties might be underestimated.

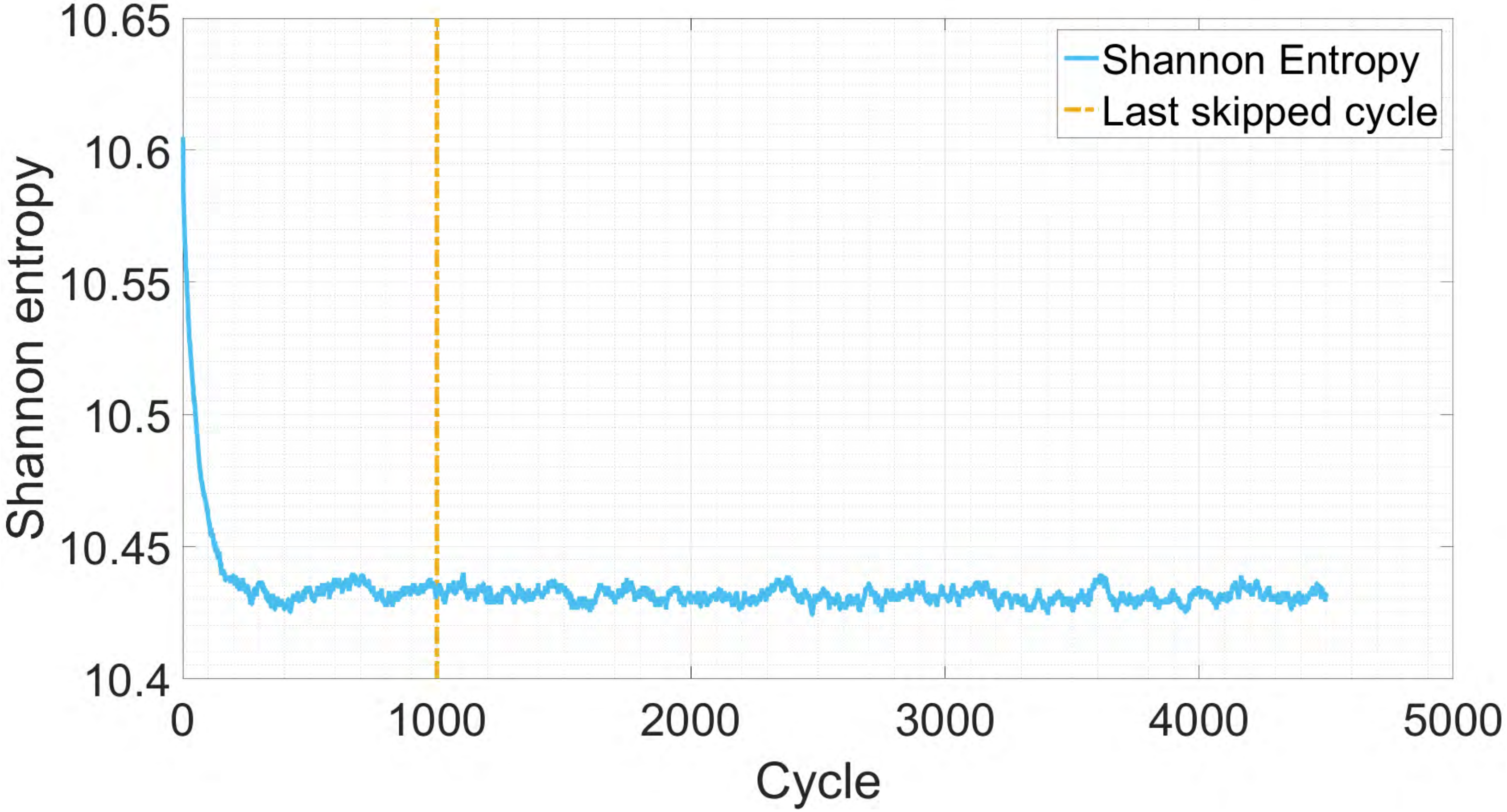
# Techniques for examining source convergence
(264 pins x 24 axial nodes = 6336 tally regions)

- Relative difference
- Shannon entropy stabilization
- $L_\infty$, $L_1$, and $L_2$ norms
- Center of Mass (COM)
- Cycle-to-cycle correlation via *replication*

# NAC, NSK, and NPS parametric analyses

## - Based on the single assembly model -

# Shannon entropy



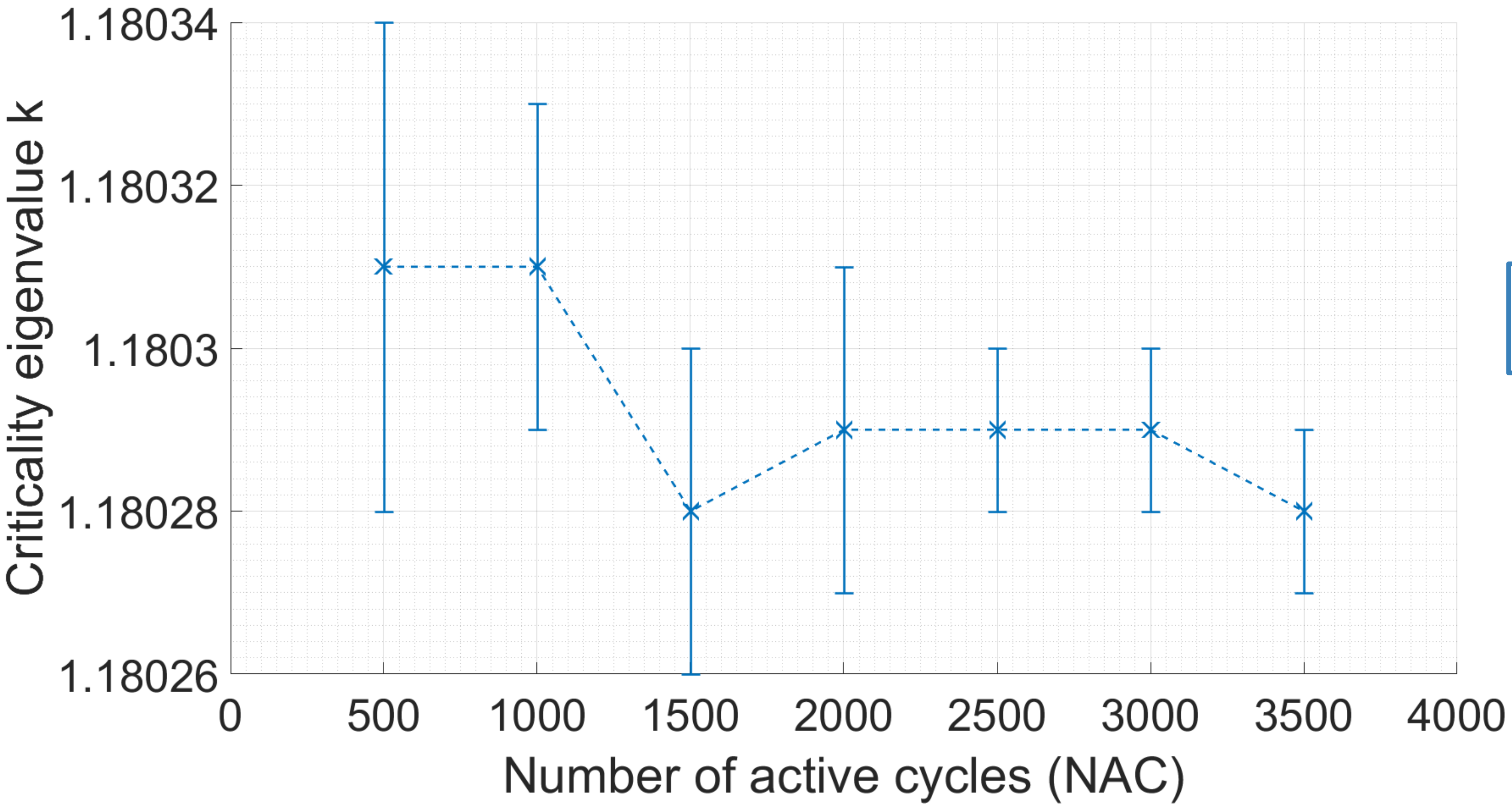

**NSK=1000, NPS=$10^6$** → ~158 *particles per tally region*

$$H = -\sum_{i=1}^{m} S_i \log_2(S_i)$$

where $m$ is the number of subregions, and $S_i$ is the ratio of fission neutrons newly generated in $i^{th}$ subregion and the total number of fission neutrons in the previous generation.

# $k_{eff}$: variation as a function NAC

NAC

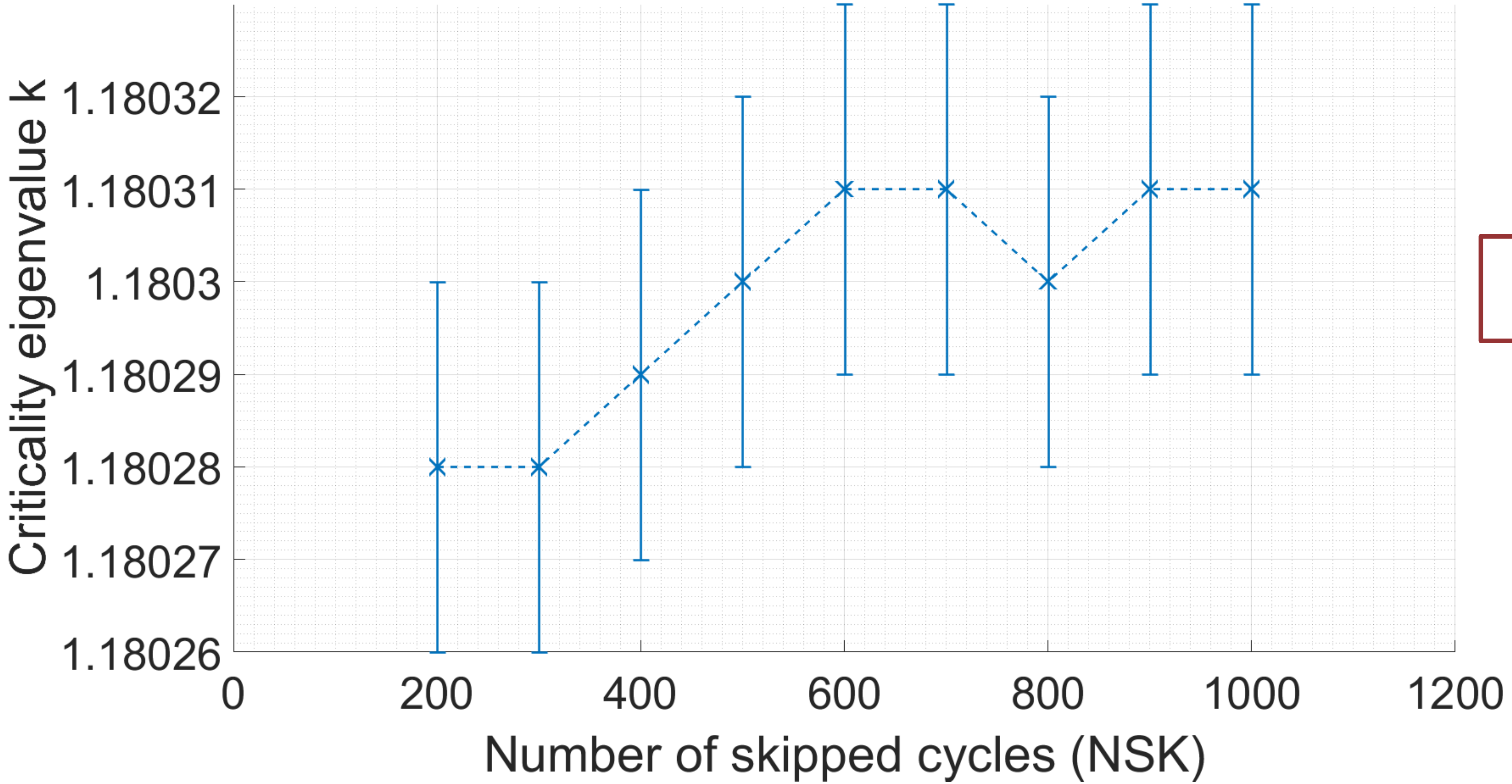



NSK=1000, NPS=$10^6$

**Note:** eigenvalue oscillates within the MCNP calculated uncertainties

# $k_{eff}$: variation as a function NSK

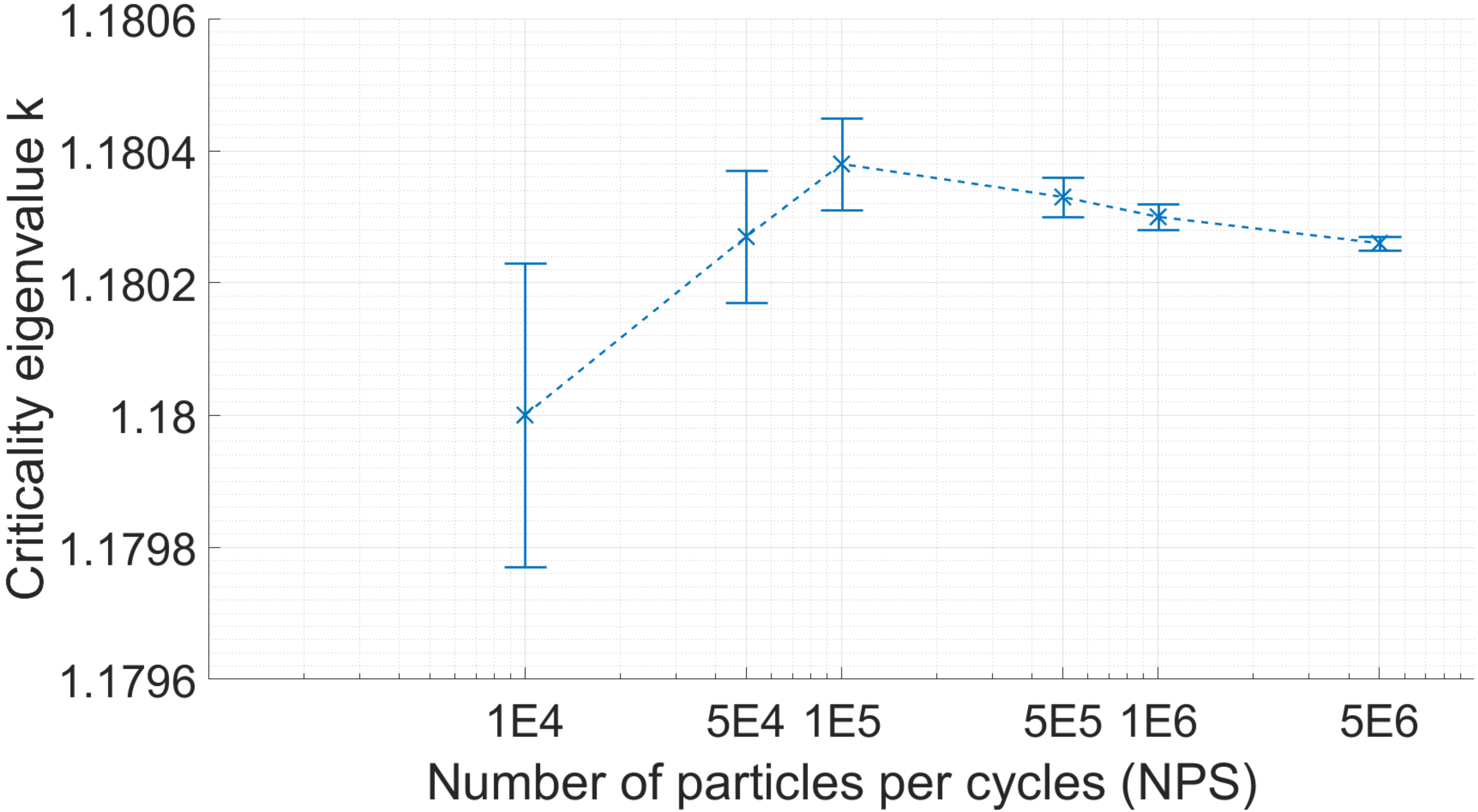
**NSK**




**NAC=1000, NPS=$10^6$**

# $k_{eff}$: variation as a function NPS

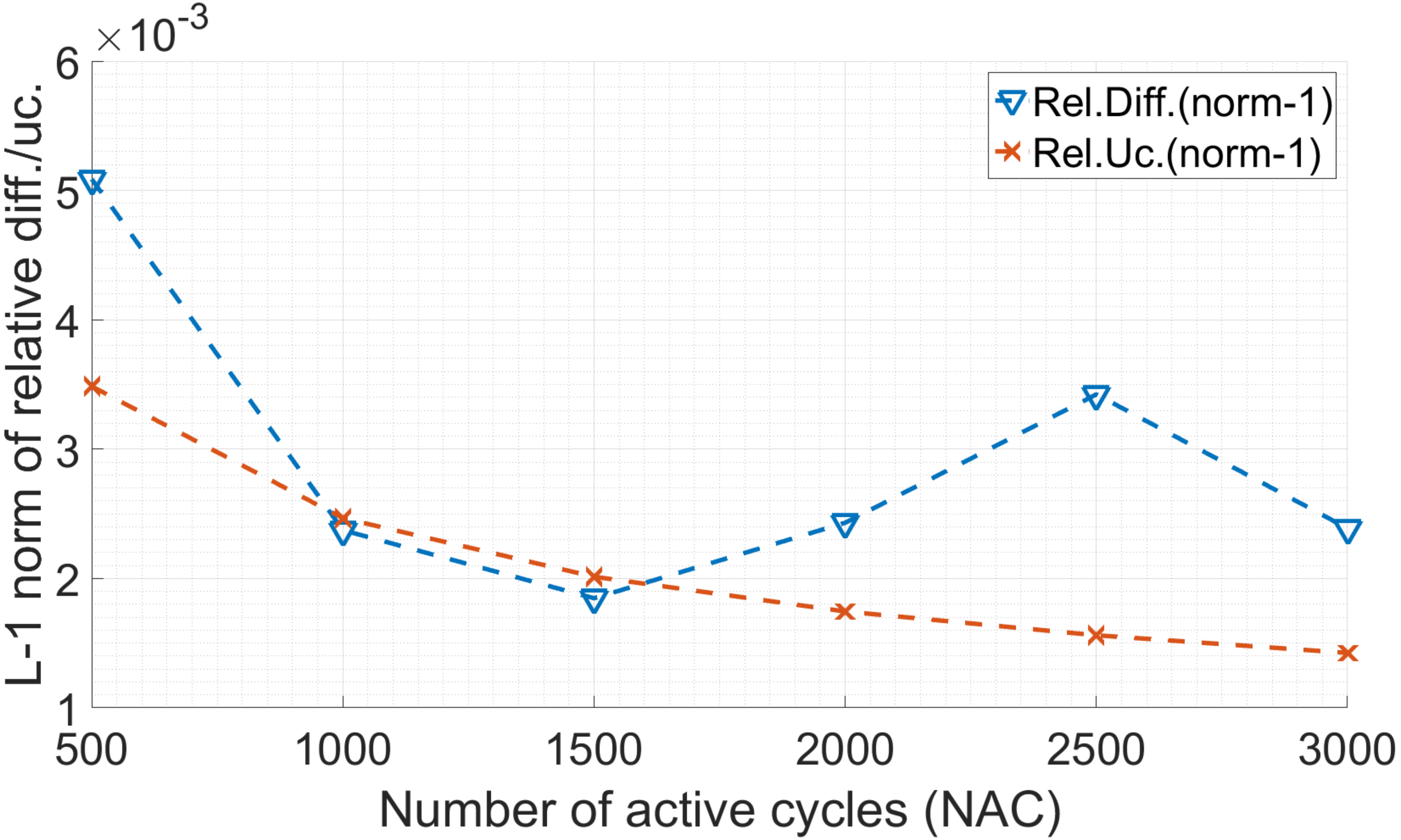
NPS




NAC=1000, NSK=500

# Fission density: $L_1$-norms of relative differences and uncertainties

**NAC**




**NSK=1000, NPS=$10^6$**

$$(Rel.Diff.)_{i,n} = \frac{S_{i,n} - S_{i,md}}{S_{i,md}}$$

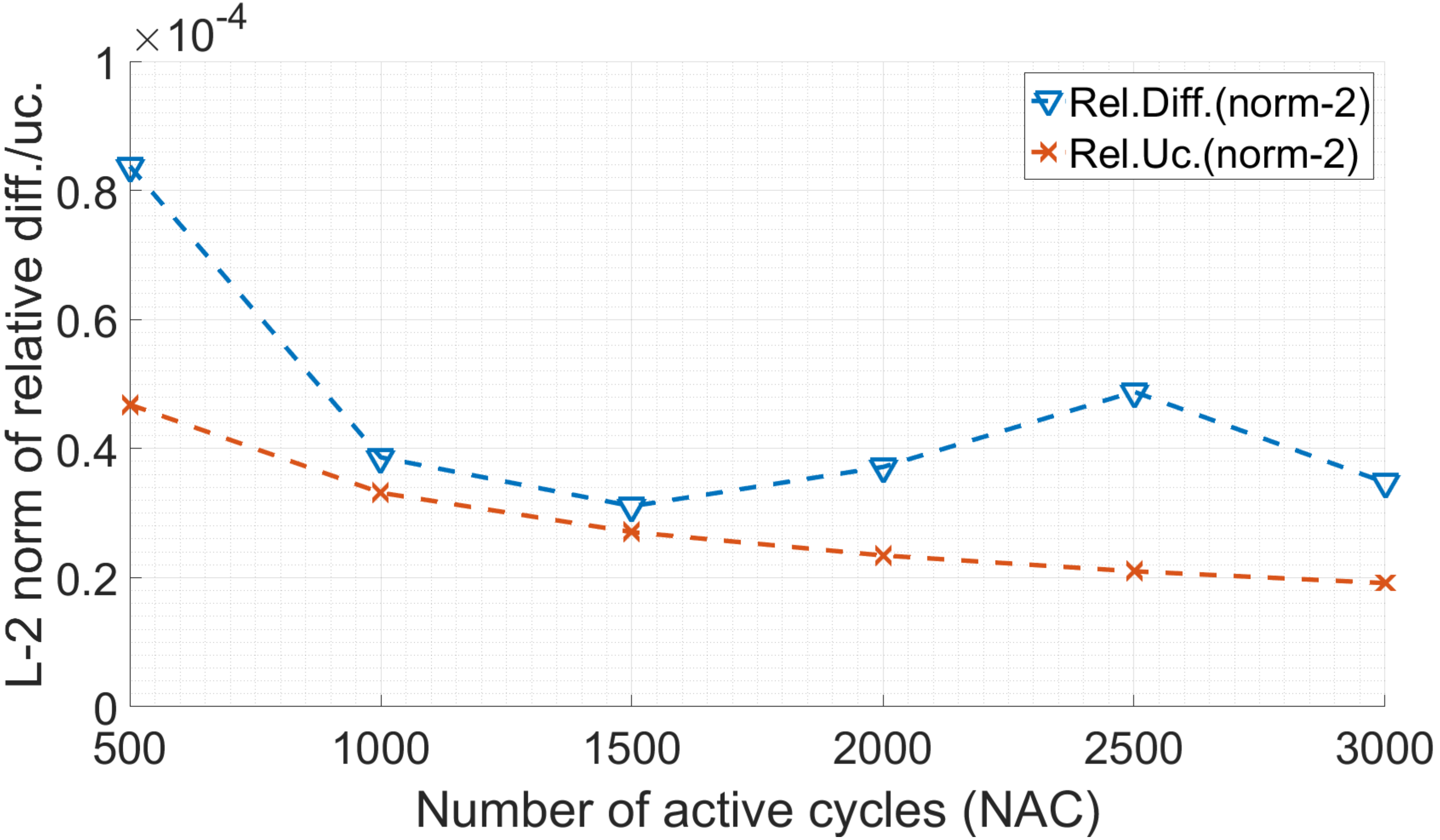
where $S_{i,n}$ and $S_{i,md}$ indicate the fission density value of the $i^{th}$ tally for the $n^{th}$ and the most-detailed ($md$) cases respectively.

$$(L_1 - norm)_n = \frac{1}{N_t} \sum_{i=1}^{N_t} |X_i|$$

where $N_t$ is the number of tally regions, and $X$ is a vector containing the relative differences (blue line) or the relative uncertainties (orange line).

# Fission density: $L_2$-norms of relative differences and uncertainties

**NAC**



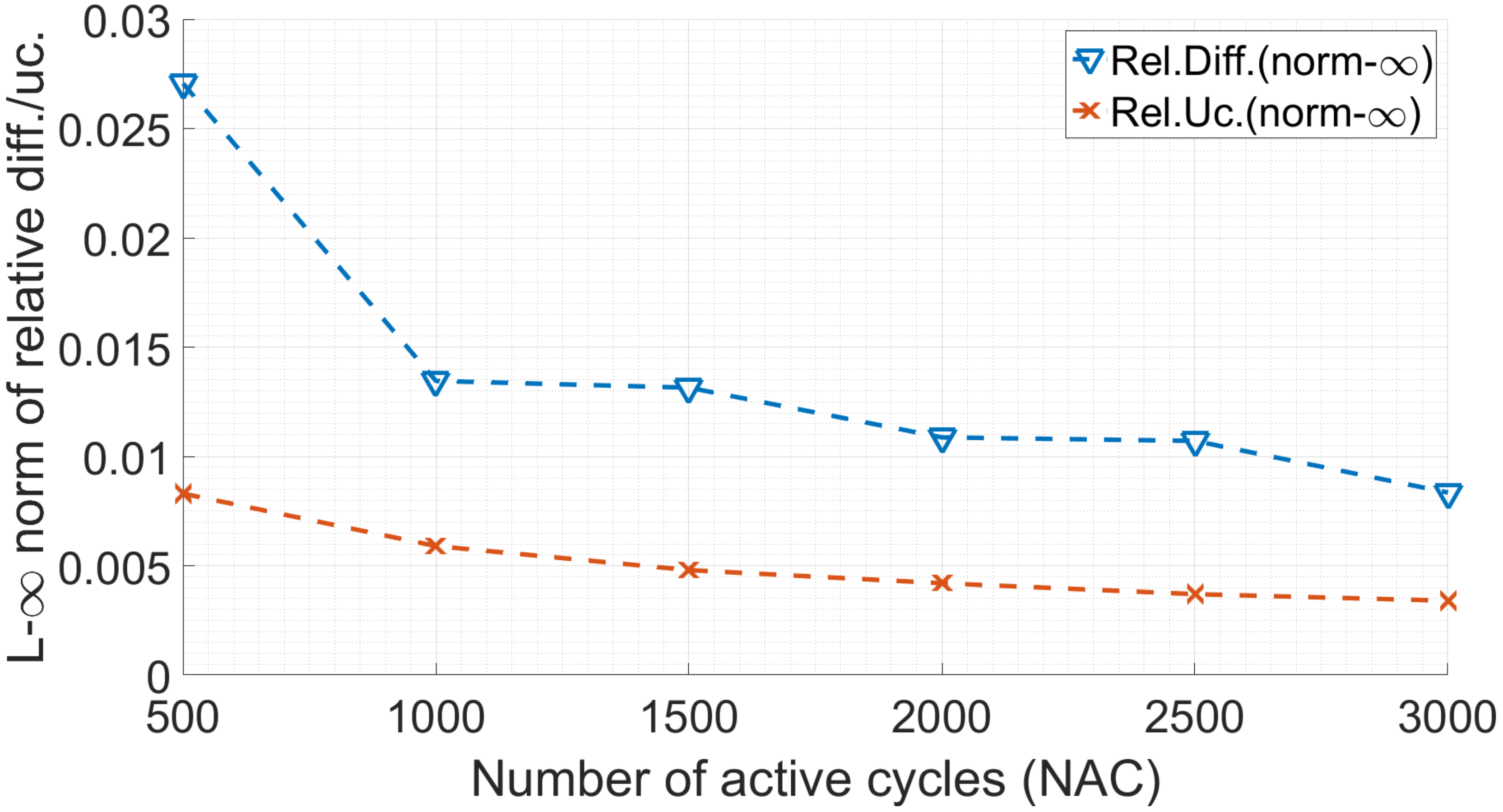

**NSK=1000, NPS=$10^6$**

$$(L_2 - norm)_n = \frac{1}{N_t}\sum_{i=1}^{N_t}\sqrt{(X_i - \bar{X})^2}$$

where $N_t$ is the number of tally regions, and $X$ is a vector containing the relative differences (blue line) or the relative uncertainties (orange line).

# Fission density: $L_\infty$-norms of relative differences and uncertainties

**NAC**




**NSK=1000, NPS=$10^6$**

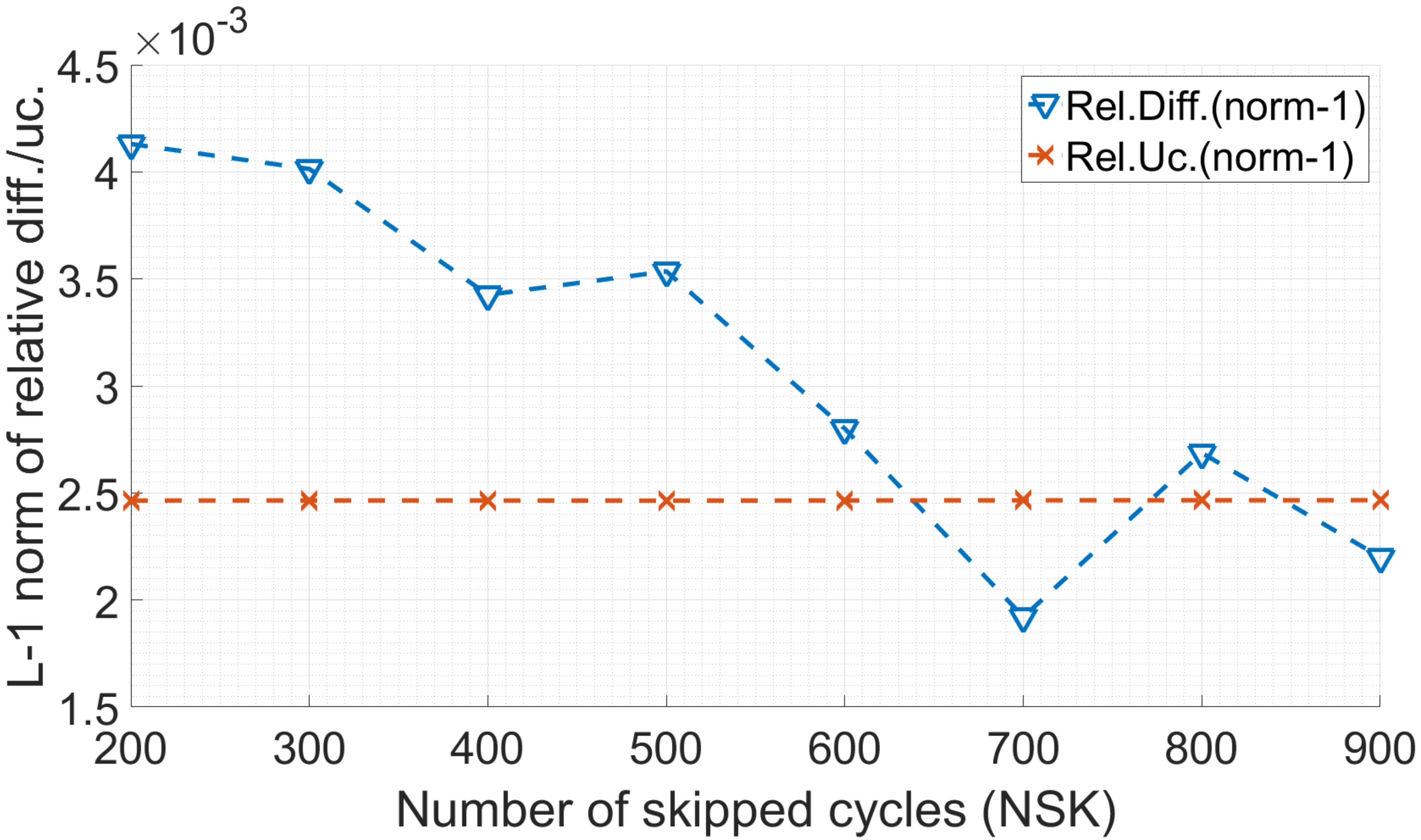
$$(L_\infty - norm)_n = \max_i S_{i,n}$$

where $N_t$ is the number of tally regions, and $X$ is a vector containing the relative differences (blue line) or the relative uncertainties (organe line).

# Fission density: $L_1$-norms of relative differences and uncertainties

**NSK**

**NAC=1000, NPS=$10^6$**




$$(Rel.Diff.)_{i,n} = \frac{S_{i,n} - S_{i,md}}{S_{i,md}}$$

where $S_{i,n}$ and $S_{i,md}$ indicate the fission density value of the $i^{th}$ tally for the $n^{th}$ and the most-detailed ($md$) cases respectively.

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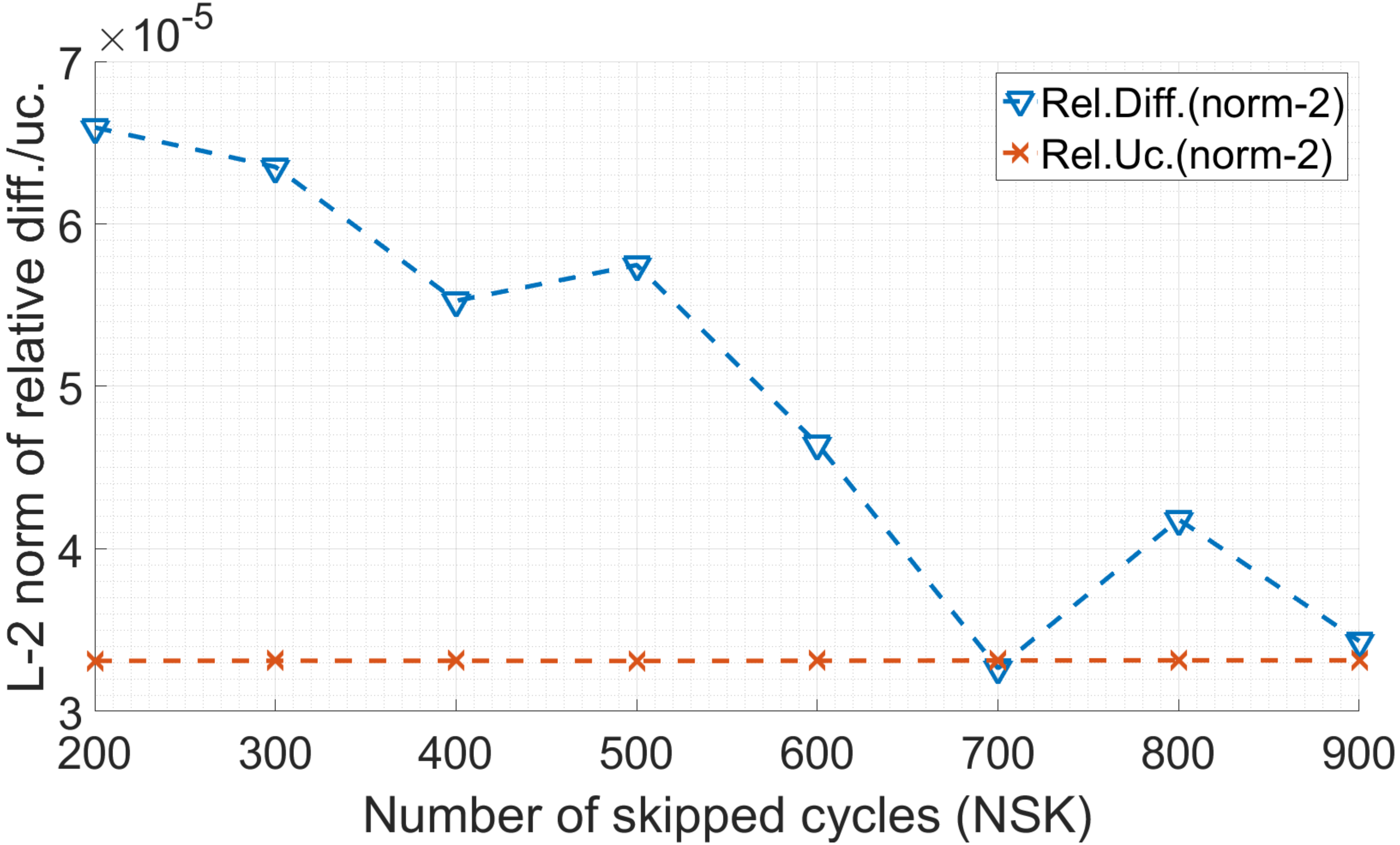
# Fission density: $L_2$-norms of relative differences and uncertainties

**NSK**




**NAC=1000, NPS=$10^6$**

$$(L_2 - norm)_n = \frac{1}{N_t}\sum_{i=1}^{N_t}\sqrt{(X_i - \bar{X})^2}$$

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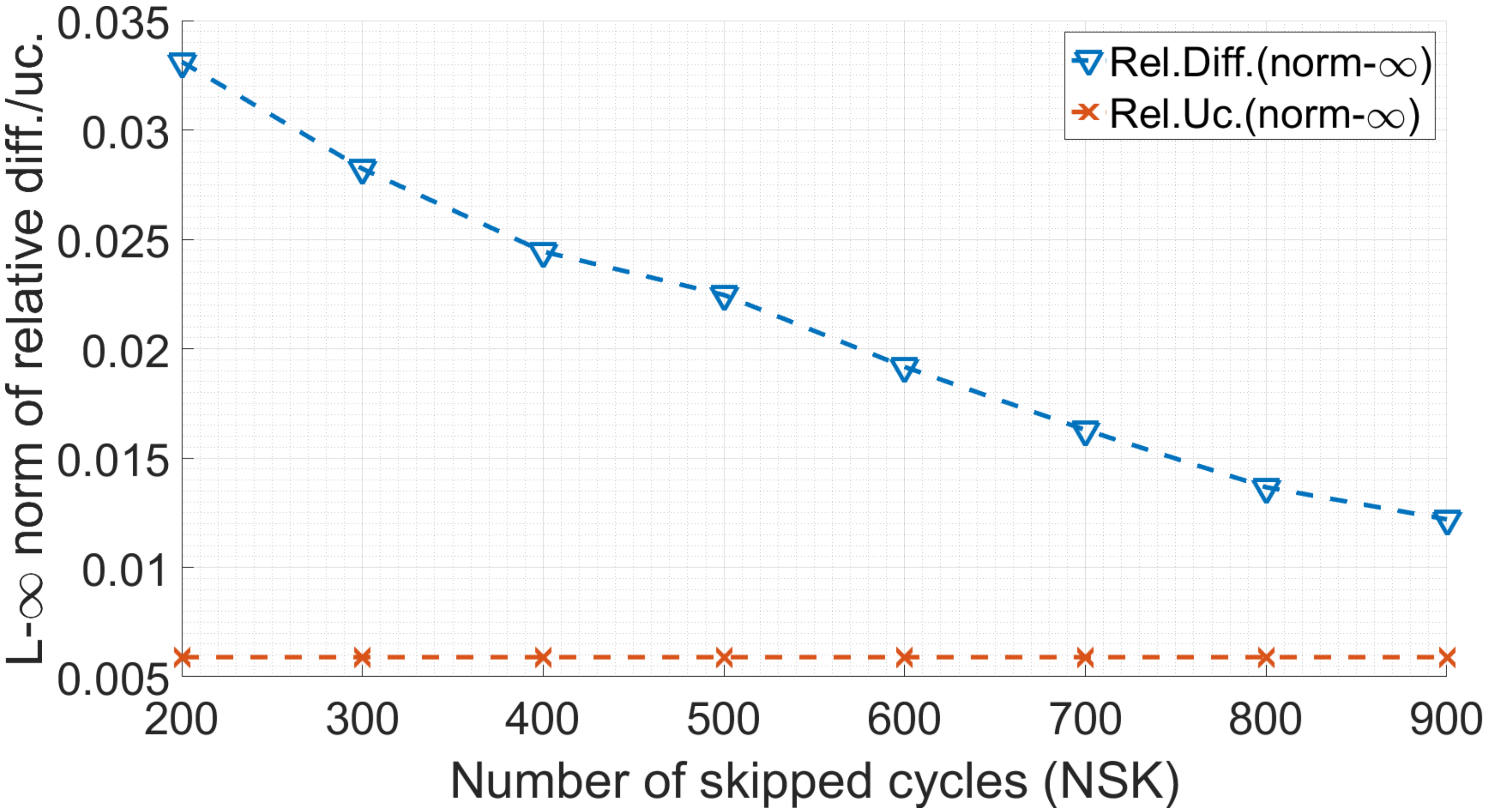
**NSK**




**NAC=1000, NPS=$10^6$**

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where $N_t$ is the number of tally regions, and $X$ is a vector containing the relative differences (blue line) or the relative uncertainties (red line).



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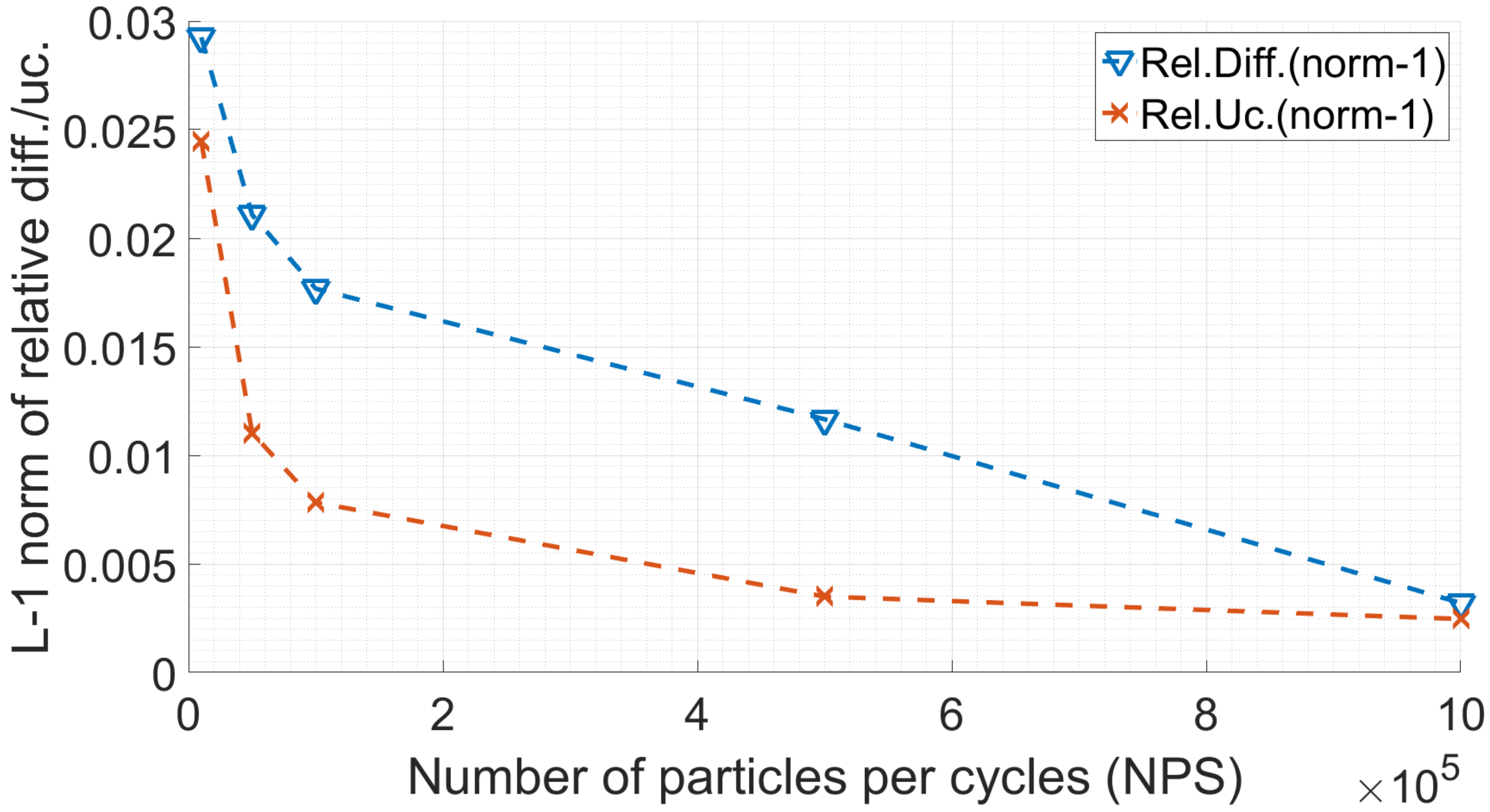
# Fission density: $L_1$-norms of relative differences and uncertainties

NPS




NSK=500,
NAC=1000

$$(Rel.Diff.)_{i,n} = \frac{S_{i,n} - S_{i,md}}{S_{i,md}}$$

where $S_{i,n}$ and $S_{i,md}$ indicate the fission density value of the $i^{th}$ tally for the $n^{th}$ and the most-detailed ($md$) cases respectively.

$$(L_1 - norm)_n = \frac{1}{N_t}\sum_{i=1}^{N_t}|X_i|$$

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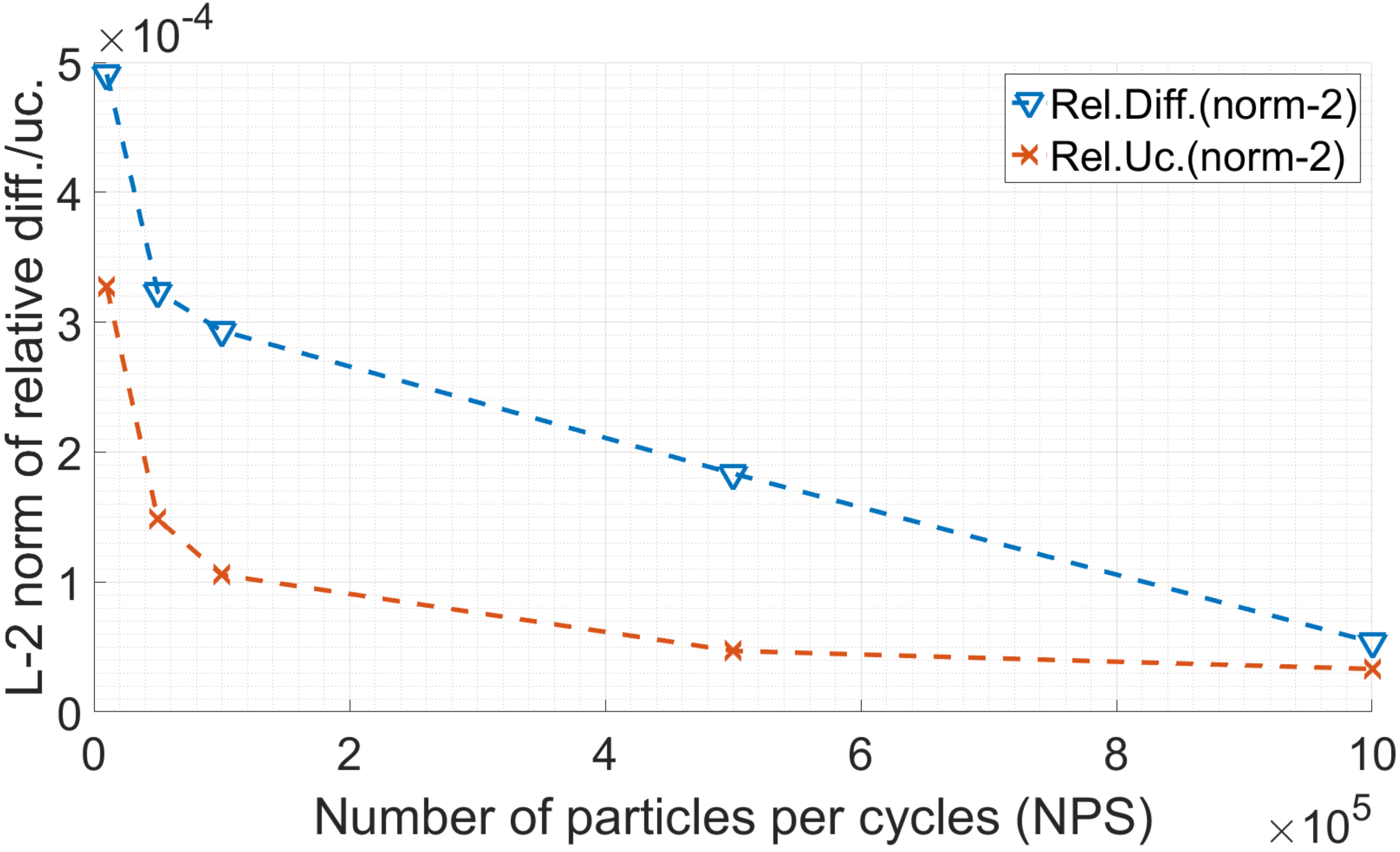
# Fission density: $L_2$-norms of relative differences and uncertainties

**NPS**



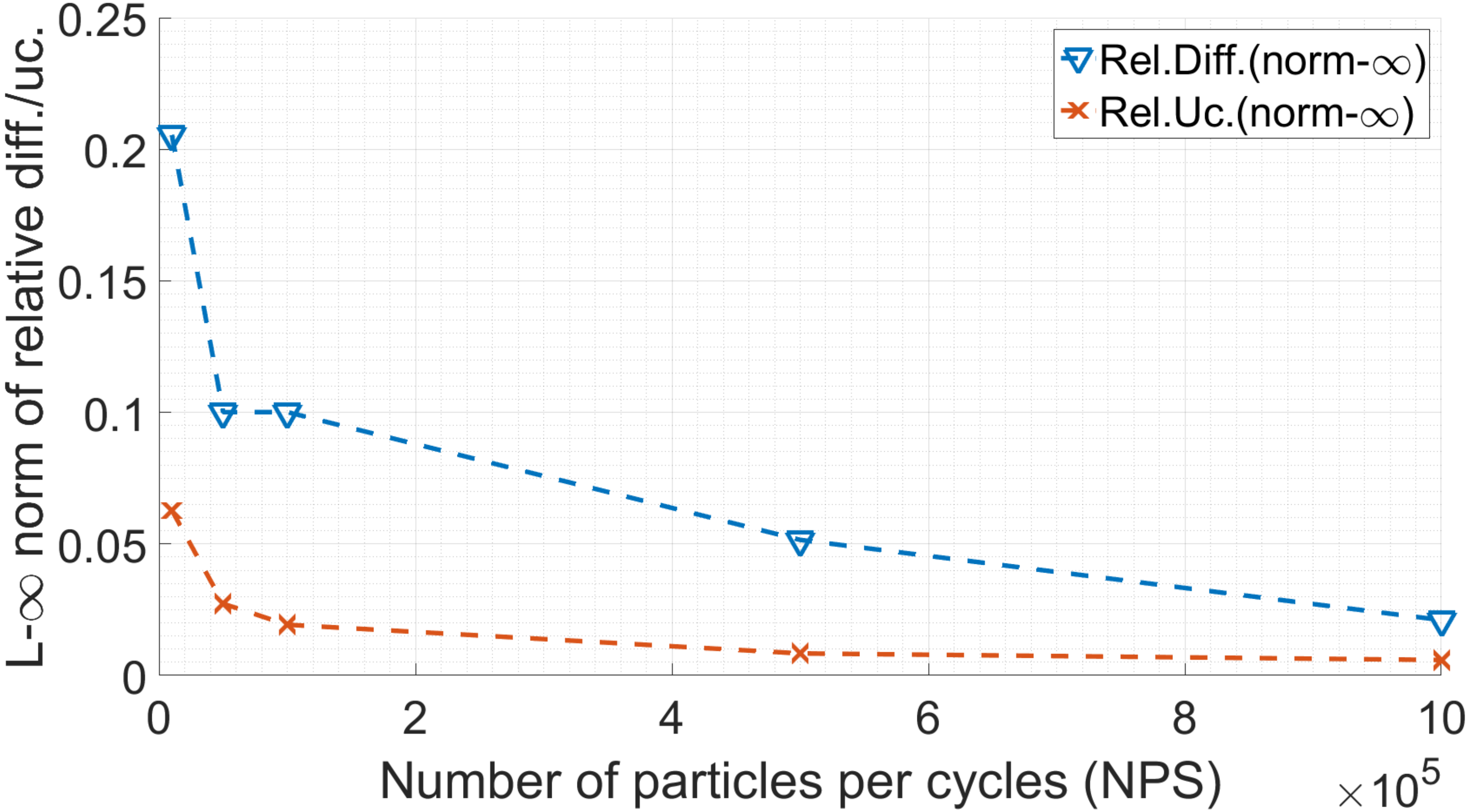

**NSK=500, NAC=1000**

$$(L_2 - norm)_n = \frac{1}{N_t}\sum_{i=1}^{N_t}\sqrt{(X_i - \bar{X})^2}$$

where $N_t$ is the number of tally regions, and $X$ is a vector containing the relative differences (blue line) or the relative uncertainties (red line).

# Fission density: $L_\infty$-norms of relative differences and uncertainties

**NPS**



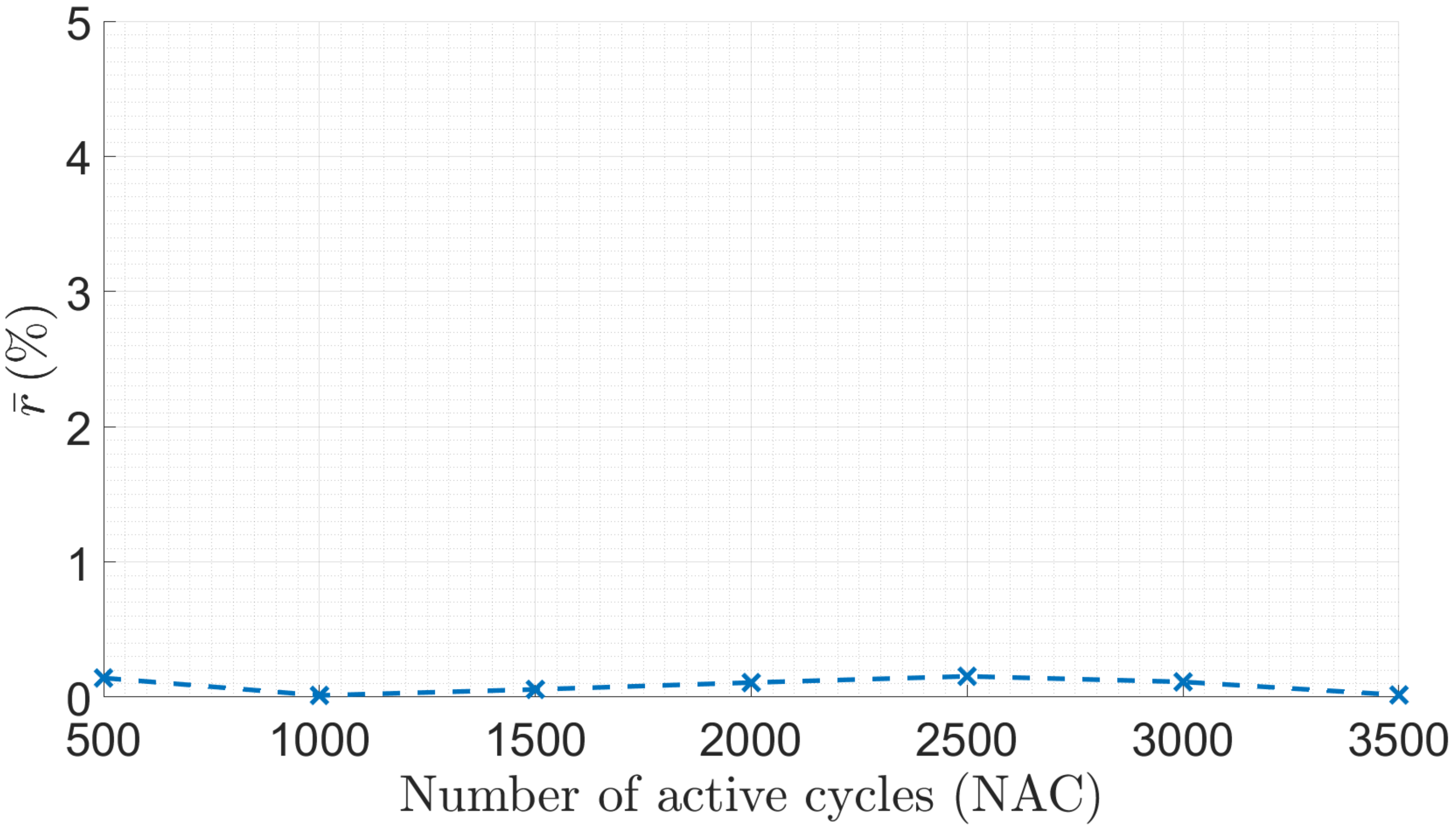

**NSK=500, NAC=1000**

$$(L_\infty - norm)_n = \max_i S_{i,n}$$

where $N_t$ is the number of tally regions, and $X$ is a vector containing the relative differences (blue line) or the relative uncertainties (red line).

# Fission density: COM distance from geometric center as a function of NAC

**NAC**




**NSK=1000, NPS=$10^6$**

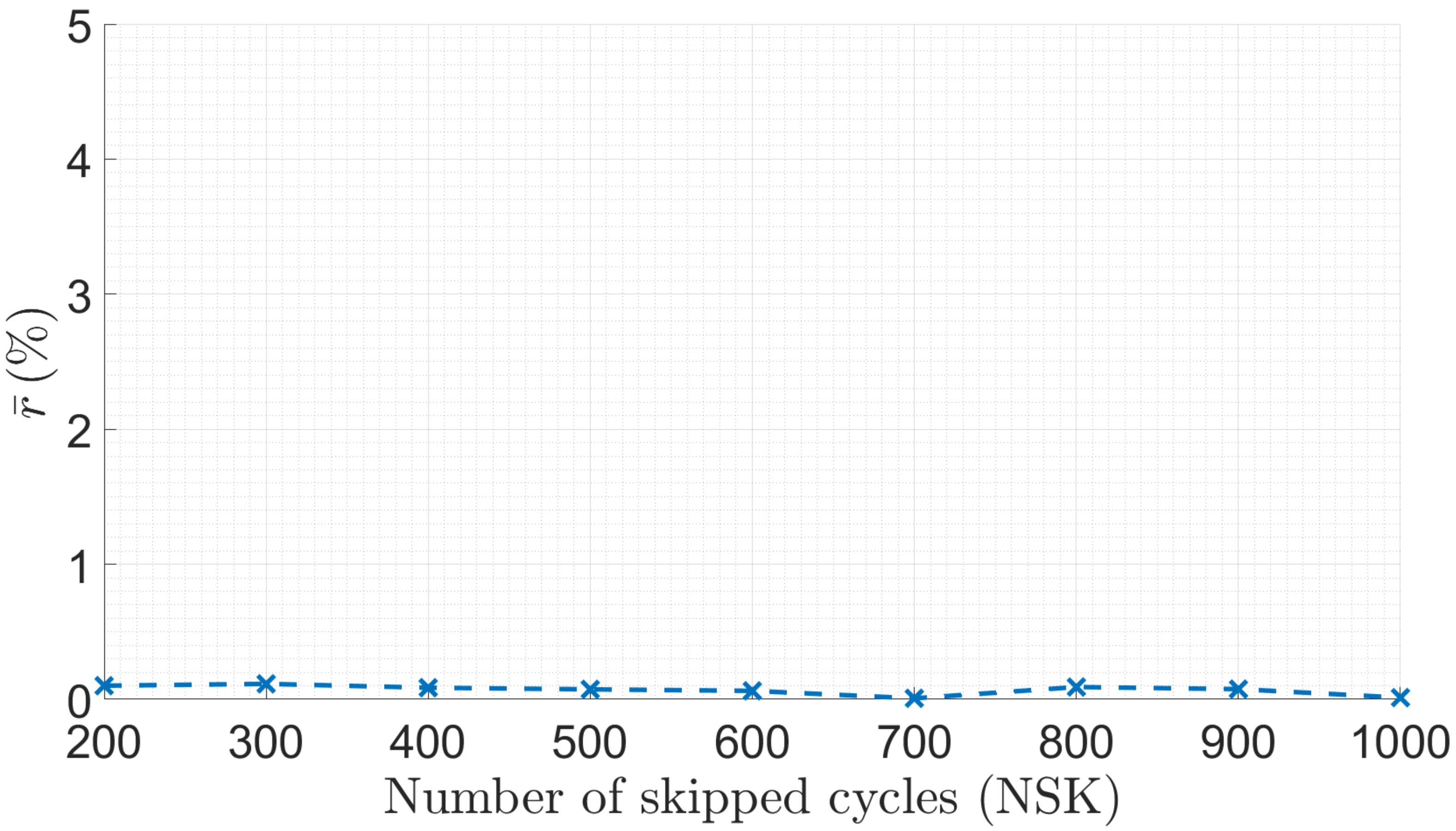
$$\overline{r_n}(\%) = \frac{100}{H/2 \cdot \sum_{i=1}^{N_t} S_{i,n}} \sum_{i=1}^{N_t} r_i S_{i,n}$$

where $r_i$ is the distance of the $i^{th}$ region from the center of the assembly, and $H$ is the active height of the fuel.

- The COM behaves like the neutron source has **converged**

# Fission density: COM distance from geometric center as a function of NSK

**NSK**




**NAC=1000, NPS=$10^6$**

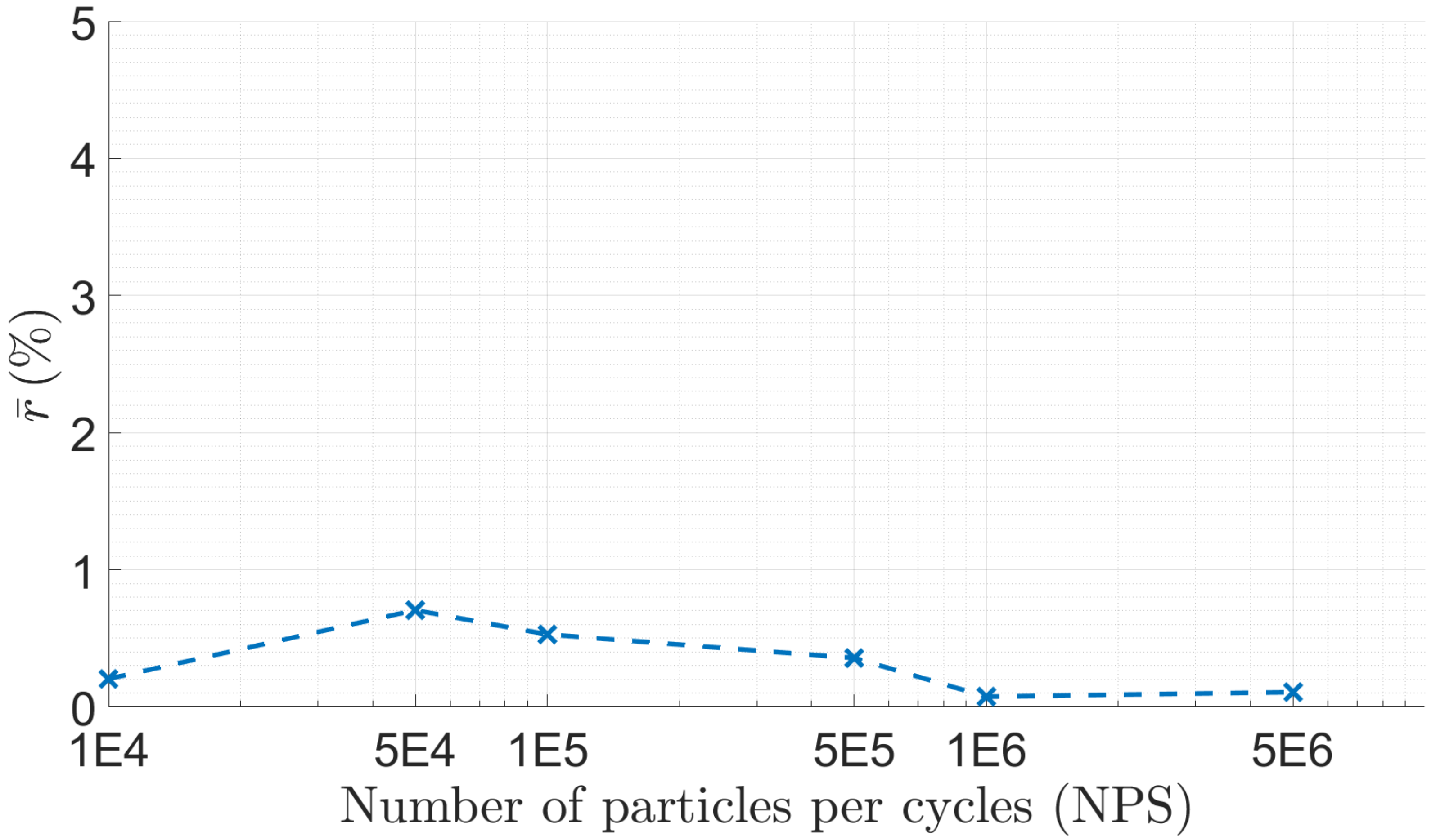
$$\bar{r}_n = \frac{100}{H/2 \cdot \sum_{i=1}^{N_t} S_{i,n}} \sum_{i=1}^{N_t} r_i S_{i,n}$$

where $r_i$ is the distance of the $i^{th}$ region from the center of the assembly, and $H$ is the active height of the fuel.

- The COM behaves like the neutron source has **converged**

# Fission density: COM distance from geometric center as a function of NPS

NPS




**NSK=500, NAC=1000**

$$\bar{r_n} = \frac{100}{H/2 \cdot \sum_{i=1}^{N_t} S_{i,n}} \sum_{i=1}^{N_t} r_i S_{i,n}$$

where $r_i$ is the distance of the $i^{th}$ region from the center of the assembly, and $H$ is the active height of the fuel.

- The COM behaves like the neutron source has **converged**

# Discussion of parametric analyses results

NSK
NAC
NPS

- Shannonentropy, COM, $L_1$, $L_2$, and $L_\infty$ behave similarly for all the parameters.

- From $L_1$, $L_2$, and $L_\infty$ norms
  it is concluded that **relative differences *on average* are higher than the statistical uncertainties.**

- From *COM* and *Shannon entropy*,
  it is conlcuded that the **fission source has converged**

- Questions?
  Is it **possible that the statistical uncertainties are underestimated?**
  **Is it caused by** the **cycle-to-cycle correlation**?

# Analysis of cycle-to-cycle correlation

- $N_r$=50 replications of a MCNP run with NSK=300, NAC=500, and NPS=$10^6$ are performed.

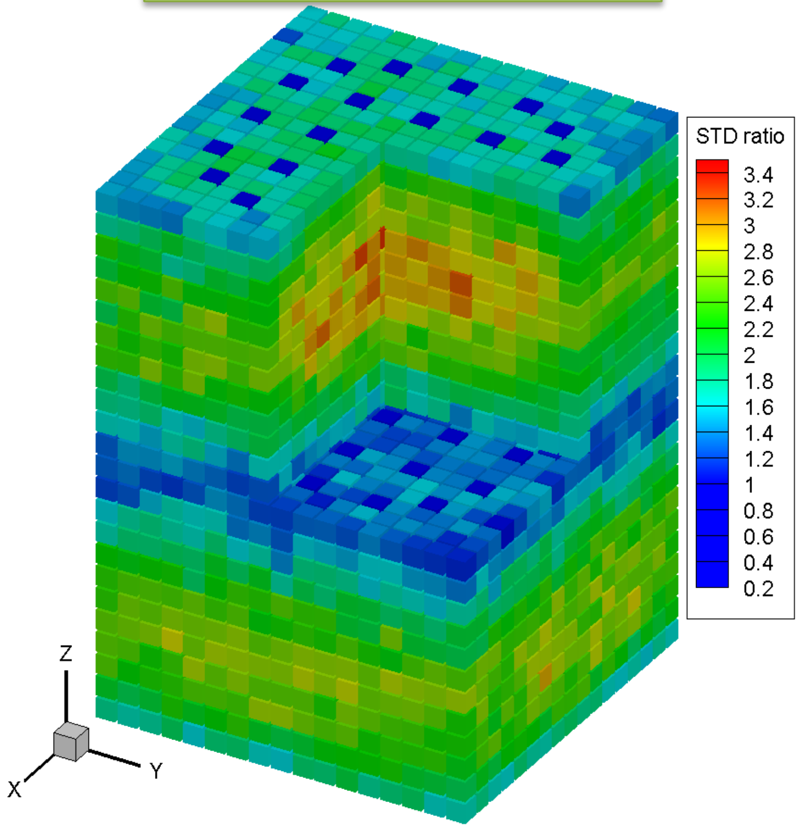
- Calculated the ratios of "actual" to MCNP statistical uncertainties, i.e., $f_{\sigma,i} = \frac{\sigma_{actual}}{\sigma_{MCNP}} = \frac{\sqrt{\frac{1}{N_r-1}\sum_{j=1}^{N_r}(S_{i,j}-\bar{S}_i)^2}}{\sigma_{MCNP}}$

- MCNP significantly **underpredicts uncertainties.**

- The weighted average of $f_\sigma$ is

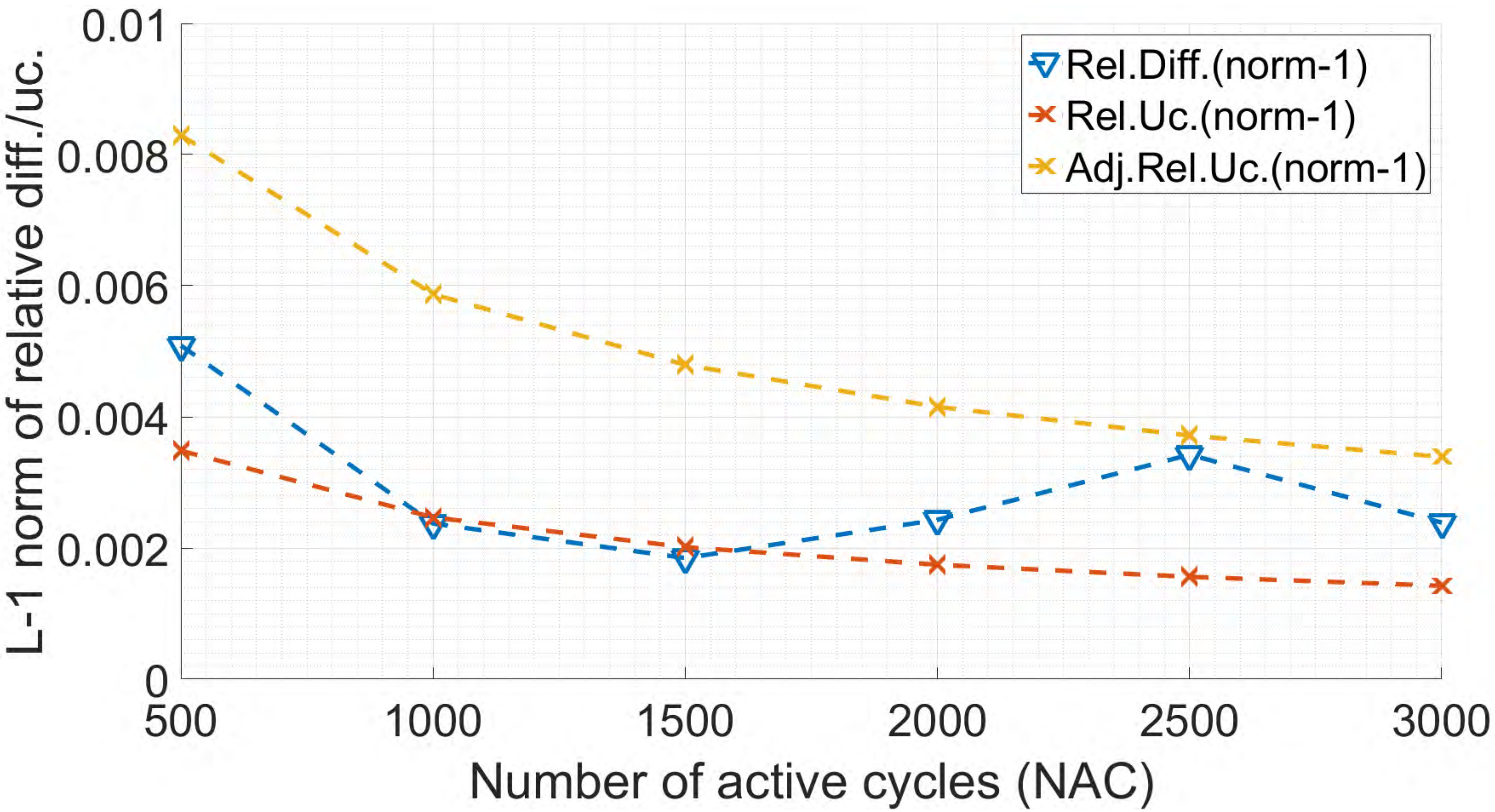
$$f_{\sigma,wgt} = \frac{\sum_{i=1}^{n} f_{\sigma i} S_i}{\sum_{i=1}^{n} S_i} = 2.28$$

$f_\sigma$ distribution



# $f_{\sigma,wgt}$ adjusted norms analysis

**NAC**

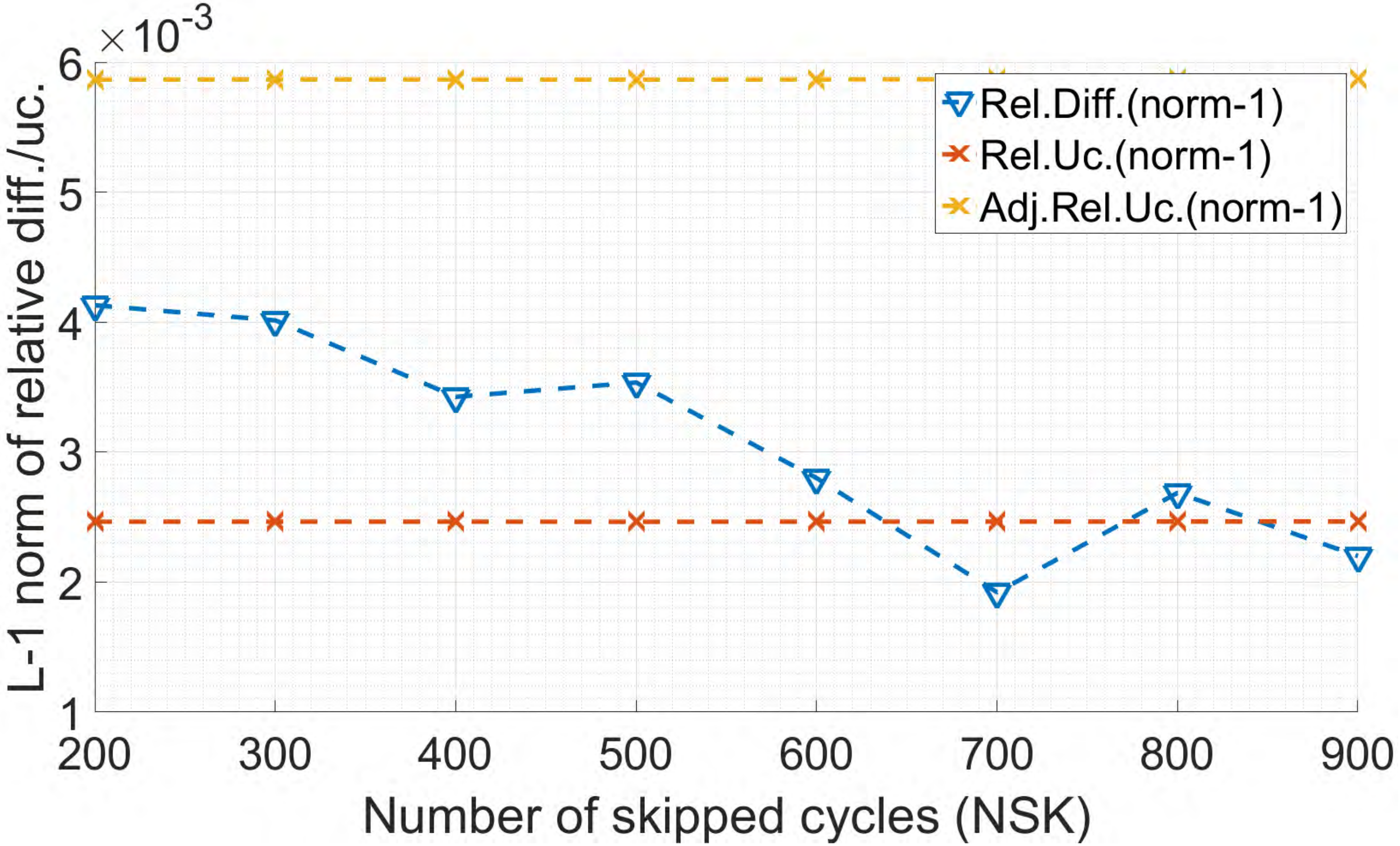



**NSK=1000, NPS=$10^6$**

Statistical uncertainties used to calculate norms factor in the $f_{\sigma,wgt}$ correction due to cycle-to-cycle correlation.

# $f_{\sigma,wgt}$ adjusted norms analysis

**NSK**

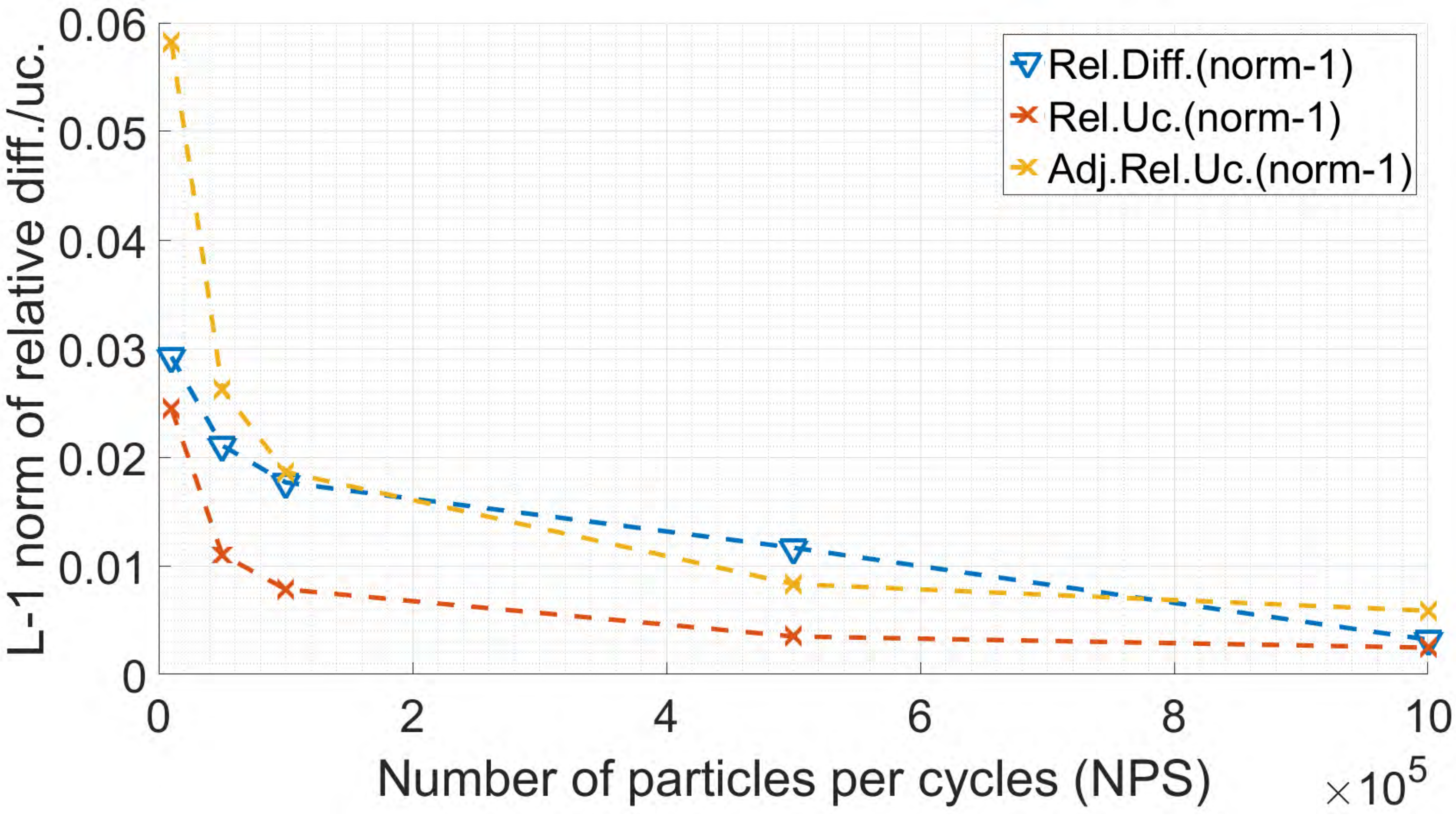



**NAC=1000, NPS=$10^6$**

Statistical uncertainties used to calculate norms factor in the $f_{\sigma,wgt}$ correction due to cycle-to-cycle correlation.

# $f_{\sigma,wgt}$ adjusted norms analysis

**NPS**




**NSK=500, NAC=1000**

Statistical uncertainties used to calculate norms factor in the $f_{\sigma,wgt}$ correction due to cycle-to-cycle correlation.

# MCNP reference Single assembly – eigenvalue parameters

- Based on this study, we selected:

  **NSK=500, NAC=1000, and NPS=$10^6$**

- This set was chosen for achieving

  **relative statistical uncertainties < 1%**

  for fission density tallies

# MCNP Full-cask model – Eigenvalue parameters

- **NAC and NSK are kept constant** due to the assemblies' uncoupling caused by absorber panels.

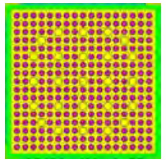
- NPS should be scaled by a factor of 32 , but NPS= $32 \cdot 10^6$ is computationally prohibitive.

- Therefore, we have used a reasonable NPS of $10^5$ per assembly, **i.e., $3.2 \cdot 10^6$ for the full cask**.

# Comparison of RAPID to MCNP reference models

## - Single assembly & full cask models -

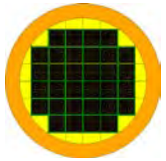
# RAPID vs. MCNP – <u>Single assembly</u> model



- RAPID calculated and MCNP system eigenvalue ($k_{eff}$) and pin-wise, axially-dependent fission density distribution, i.e, **6,336 tallies**, are compared.
- **Significant speedup** is obtained using RAPID on just a single computer core.

| Case | MCNP | RAPID |
|---|---|---|
| $k_{eff}$ | 1.18030 (± 2 pcm) | 1.18092 |
| $k_{eff}$ relative difference | - | 53 pcm |
| Fiss. density adjusted rel. uncertainty | 0.48% | - |
| Fission density relative diff. | - | 0.65% |
| Computer | 16 cores | 1 core |
| Time | 666 min (11.1 hours) | 0.1 min (6 seconds) |
| Speedup | - | 6,666 |

# RAPID vs. MCNP – Full cask model

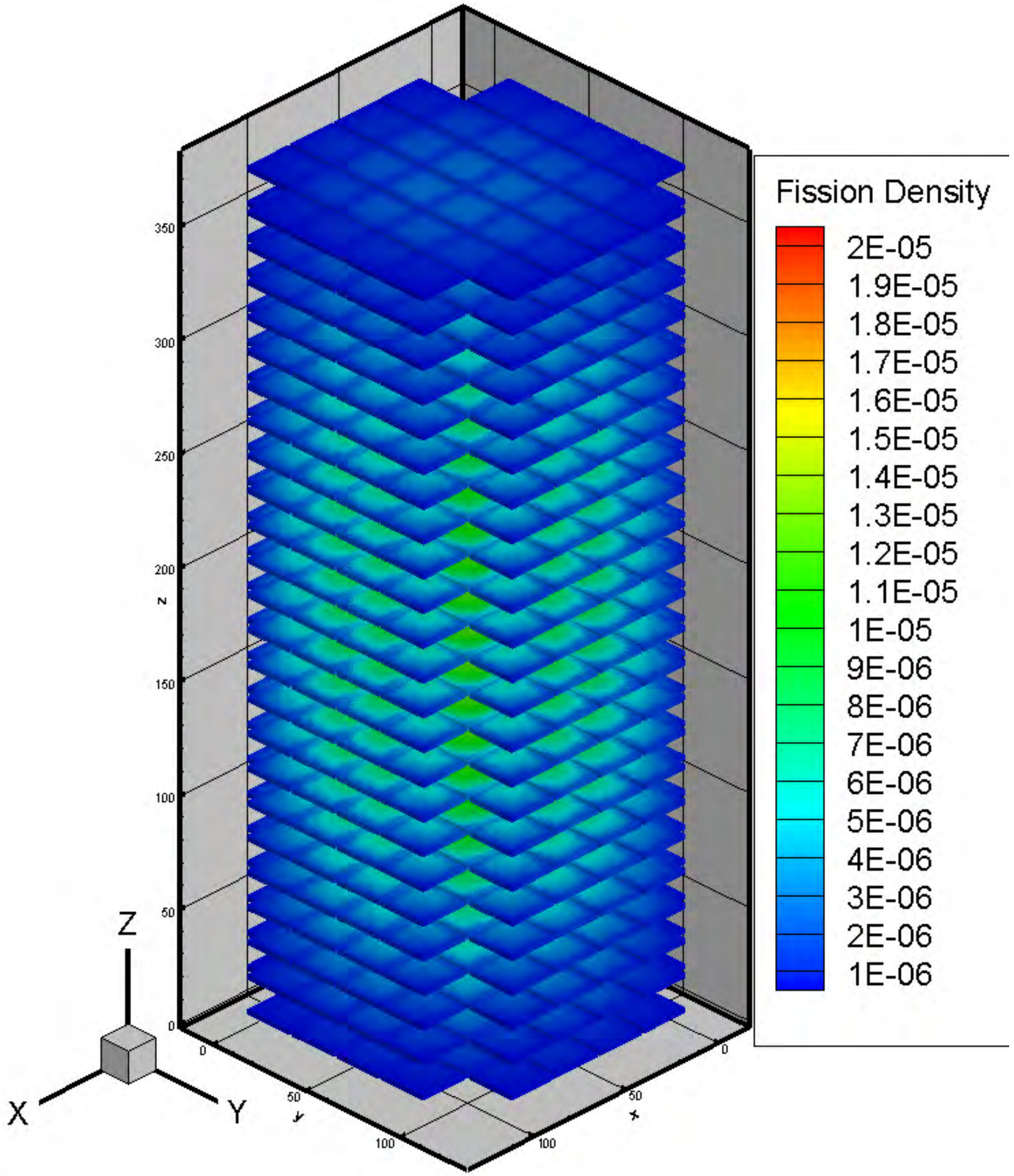


- RAPID calculated and MCNP system eigenvalue ($k_{eff}$) and pin-wise, axially-dependent fission density distribution, i..e, **202,752** tallies (for ~15.8 particles per tally region), are compared.

- **The speedup increases with the dimension of the model.**

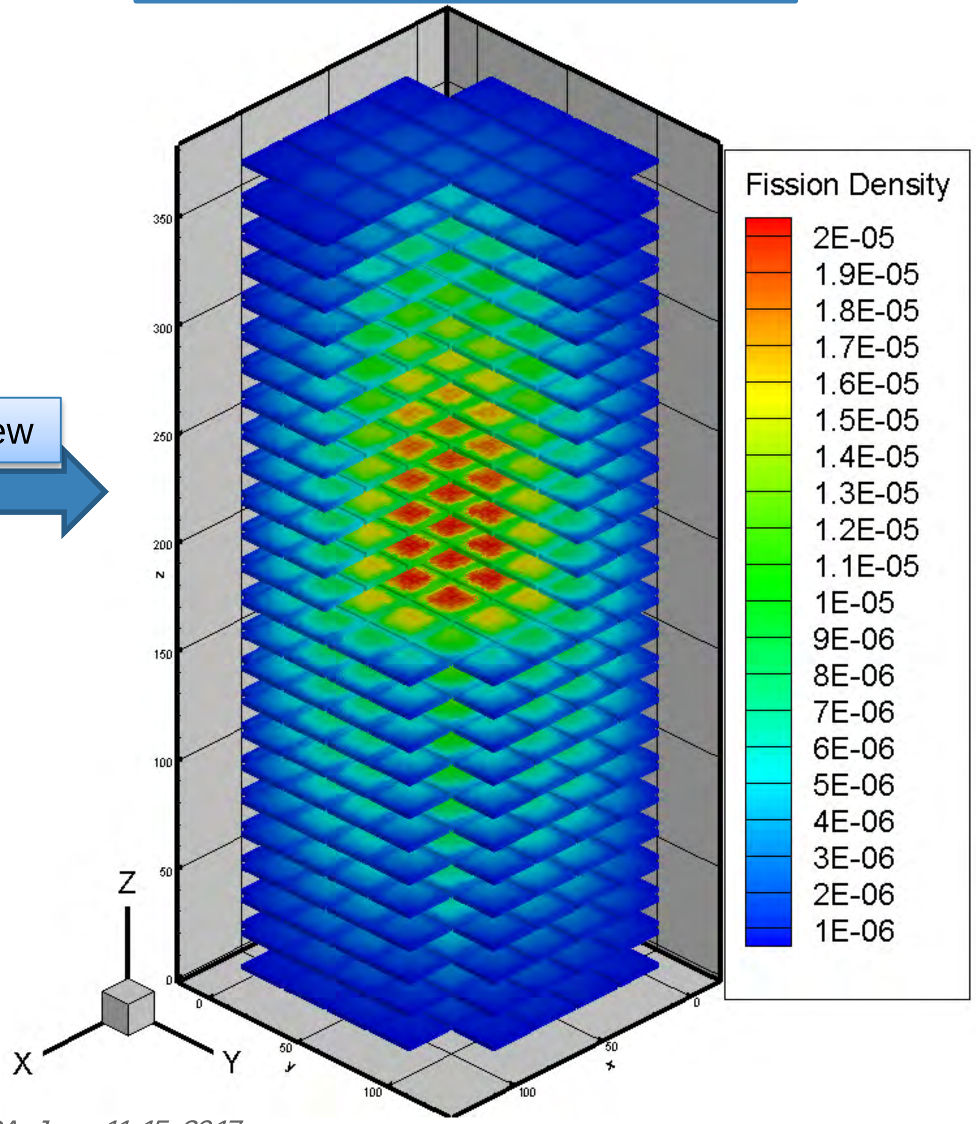
| Case | MCNP | RAPID |
|---|---|---|
| $k_{eff}$ | 1.14545 (± 1 pcm) | 1.14590 |
| Relative Difference | - | 39 pcm |
| Fission density rel. uncertainty | 1.15% | - |
| Fission density relative diff. | - | 1.56% |
| Computer | 16 cores | 1 core |
| Time | 13,767 min (9.5 days) | 0.585 min (35 seconds) |
| Speedup | - | 23,533 |

# GBC-32 3D fission density distribution




Inside view

**With a quarter Blanked**



# Concluding remarks

▷ It is demonstrated that RAPID can obtain accurate pin-wise, axially-dependent fission source distribution and $k_{eff}$ in a **whole UNF c**ask in **real time (seconds)**.

▷ The RAPID MRT algorithm is able t**o overcome the main issues** related to Monte Carlo eigenvalue calculations such as source convergence and cycle-to-cycle correlation.

# Ongoing and future work

- External dose/detector response calculation has been implemented into the RAPID system using the PENTRAN-calculated importance function methodology. (presented at Work presented at the ANTPC conference in Santa Fe, New Mexico, Sep 25-30, 2016.)

- Developingan algorithm for direct calculation of response coefficients

- Initiated discussions with a nuclear utility for perform *experimental benchmarking* of RAPID based on measurements of the cask surface dose

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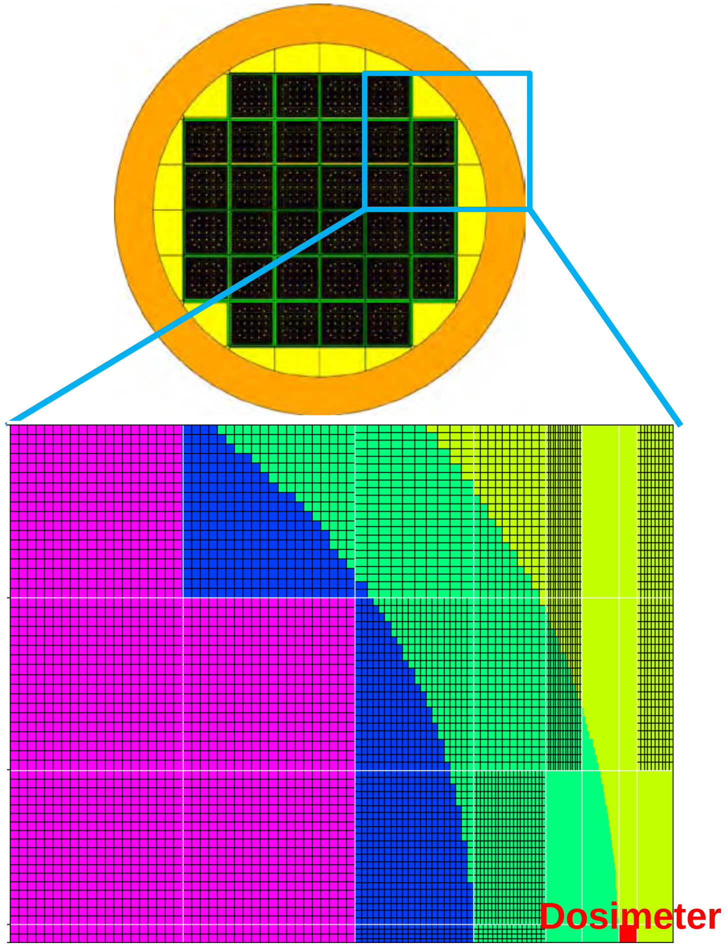

# Questions?

**Thanks**